The London School of Economics and Political Science

Time-Varying Liquidity and Profitability of Hedge Funds

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Declaration:

I certify that the thesis I have presented for examination for the PhD degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it).

I warrant that this authorization does not, to the best of my belief, infringe the rights of any third party.

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Abstract

The hedge fund industry has grown to be one of the most important segments of the financial services industry. Hedge funds are known for employing highly dynamic trading strategies and investing in illiquid assets to increase their profitability. In this thesis, we develop and test the models that capture the time-varying nature of liquidity and profitability of hedge funds.

The thesis begins with the study of the liquidity of hedge funds' investments. We propose a method for determining the factors that affect the (unobservable) liquidity of hedge fund investments. We find substantial evidence of time variation in the liquidity of hedge fund returns, and that this time variation can be predicted with readily available data.

We then examine the impact of market dispersion on the profitability of hedge funds. Market dispersion is measured by cross-sectional volatility, that is, the standard deviation across all asset returns in one time period. We exploit the information held in the cross-sectional dispersion of equity returns and find that market dispersion and the performance of hedge funds are positively related across all equity-oriented hedge funds.

Furthermore, to gain a better understanding of hedge fund risk, in the third chapter we assess the empirical success of Fung-Hsieh, Fama-French and Statistical Factor Models for explaining hedge fund returns and compare their explanatory power for the cross section of hedge fund returns.

In the final chapter, we introduce a general and flexible framework for hedge fund performance evaluation and asset allocation: stochastic dominance theory. Our approach utilizes statistical tests for stochastic dominance to evaluate the performance of hedge funds. To illustrate the method's ability to work with non-normal distributions, we form hedge fund portfolios by using stochastic dominance criteria and examine the out-of-sample performance of these hedge fund portfolios. Compared to performance of portfolios of randomly selected hedge funds and mean-variance efficient hedge funds, our results show that fund selection method based on stochastic dominance criteria greatly improves the performance of hedge fund portfolios.

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Introduction

The hedge fund industry has grown to be one of the most important segments of the financial services industry. Hedge funds are known for employing highly dynamic trading strategies and investing in illiquid assets to increase their profitability. In this thesis, we develop and test the models that capture the time-varying nature of liquidity and profitability of hedge funds.

The thesis consists of four chapters. The first chapter focuses on the liquidity of hedge funds' investments. It is of great interest both to hedge fund investors and to market regulators. We propose a method for determining the factors that affect the (unobservable) liquidity of hedge fund investments. Our method exploits the link between illiquidity and serial correlation in hedge fund returns established by Getmansky, Lo and Makarov (2004), and does not require information on the actual positions taken by the hedge fund, nor even the `style' of the hedge fund; we use only the returns reported by the hedge fund and other easily observed information. Using a panel of monthly returns on over 600 individual hedge funds, we find significant evidence of time variation in the degree of liquidity of hedge fund investments. Broadly stated, hedge funds in equity-based styles, such as equity market neutral and equity hedge or non-hedge, exhibit decreases in liquidity when stock market returns are low and bond market returns are high. In contrast, hedge funds in fixed income styles, such as convertible arbitrage or fixed income, exhibit lower liquidity when equity market volatility is high, and when the fund experiences in-flows or out-flows of funds.

We then examine the impact of market dispersion on the performance of hedge funds. Market dispersion is measured by cross-sectional volatility, that is, the standard deviation across all asset returns in one time period. We exploit the information held in the cross-sectional dispersion of equity returns and find that market dispersion and the performance of hedge funds are positively related across all equity-oriented hedge funds. Containing information very different from other factors, cross-sectional volatility is an important determinant of hedge fund returns. We also find the level of hedge fund return dispersion is positively related to the level of market dispersion.

To gain a better understanding of hedge fund risk, in the third chapter we assess the empirical success of Fung and Hsieh (2004) asset based style factors, a five-factor extension of Fama-French factors and statistical factors for explaining the hedge fund returns. We document that the first two sets of factor models explain a significant part of the systematic exposure of hedge funds and that the explanatory power of the five-factor extension of Fama and French is larger than those of the Fung and Hsieh seven-factor for all categories except for Distressed Securities and Fixed Income Arbitrage. Asymptotic principal component analysis of the individual fund regression residuals reveals that there are latent factors which are not captured by Fung-Hsieh and Fama-French models.

In the final chapter, we introduce a general and flexible framework for hedge fund performance evaluation and asset allocation: stochastic dominance (SD) theory. Our approach utilizes statistical tests for stochastic dominance to compare the returns of hedge funds. We form hedge fund portfolios by using SD criteria and examine the out-of-sample performance of these hedge fund portfolios. Compared to performance of portfolios of randomly selected hedge funds and mean-variance efficient hedge funds, our results show that fund selection method based on SD criteria greatly improves the performance of hedge fund portfolio.

Chapter 1

Time-Varying Liquidity in Hedge Fund Returns

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The liquidity of hedge funds' investments is of great interest both to hedge fund investors and to market regulators. We propose a method for determining the factors that affect the (unobservable) liquidity of hedge fund investments. Our method exploits the link between illiquidity and serial correlation in hedge fund returns established by Getmansky, Lo and Makarov (2004), and does not require information on the actual positions taken by the hedge fund, nor even the 'style' of the hedge fund; we use only the returns reported by the hedge fund and other easily observed information.

Using a panel of monthly returns on over 600 individual hedge funds, we find significant evidence of time variation in the degree of liquidity of hedge fund investments. Broadly stated, hedge funds in equity-based styles, such as equity market neutral and equity hedge or non-hedge, exhibit decreases in liquidity when stock market returns are low and bond market returns are high. In contrast, hedge funds in fixed income styles, such as convertible arbitrage or fixed income, exhibit lower liquidity when equity market volatility is high, and when the fund experiences in-flows or out-flows of funds.

Keywords: Liquidity; Serial correlation; Return smoothing; Hedge funds

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1 Introduction

The liquidity of hedge funds' investments is of great interest to hedge fund investors and to market regulators. The degree of investment liquidity directly affects the liquidity that the fund can offer its investor; funds with less liquid investments require longer "lock-up" and redemption notification periods from their investors in order to avoid costly exits from illiquid investments. Regulators have expressed much concern about the liquidity of hedge funds' investments and the potential market impact of the collapse of a fund[1]. The 1998 Long Term Capital Management crisis highlighted liquidity as a particularly important source of risk for hedge funds, for both investors and regulators.

Whilst there is much interest in hedge fund liquidity, it remains an elusive concept to measure and has a variety of definitions and interpretations. Most definitions suggest, in some way, that highly liquid markets are those where it is possible to trade large quantities of the asset quickly and at low cost. Measures of liquidity for standard assets include variables such as the bid-ask spread, the volume of turnover, and possibly the depth of the best bid and ask quotes. Using these, and other proxies for liquidity, the liquidity risk in stock and bond markets has been intensively studied[2]. The liquidity risk of hedge funds has recently begun to attract attention, though the problem is complicated by the fact that the liquidity proxies used in studies of stock and bond markets are not directly applicable to hedge funds. The attention to hedge fund liquidity comes at an important time: the assets under management by hedge funds was estimated to have surpassed \$1.4 trillion at the end of 2006, a 40% increase over the previous 12 months, and approximately a 28-fold increase since 1990.

A number of recent papers have studied the returns generated by hedge funds, though

[1] As pointed out by the new Chairman of the Federal Reserve Board at the Federal Reserve Bank of Atlanta's 2006 Financial Markets Conference: "Much of the recent debate...has focused on the opacity of hedge funds to regulatory authorities and to the markets generally, which is viewed by some as an important source of liquidity risk. Liquidity in a particular market segment might well decline sharply and unexpectedly if hedge funds chose or were forced to reduce a large exposure in that segment. Concerns about hedge fund opacity and possible liquidity risk have motivated a range of proposals for regulatory authorities to create and maintain a database of hedge fund positions."

[2] See Fleming (2003), Pástor and Stambaugh (2003), Fujimoto (2004), Avramov, *et al.* (2005), Chordia, *et al.* (2005), Goldreich, *et al.* (2005) and Johnson (2006), amongst others.

none of these studies directly focus on the liquidity risk of hedge funds[3]. Hedge fund liquidity, generally considered, has various aspects. From the perspective of the investors in hedge funds, the "lock-up" and redemption notice periods are the sources of illiquidity of most concern. The lock-up period is the period that investors must wait before withdrawing their initial investment in the fund, and is usually on the order of one to eighteen months. The redemption notice period is the notification period required before investors can redeem their shares, which is usually one to three months. The main source of illiquidity from the perspective of the hedge fund manager is the liquidity of the fund's investments. As mentioned above, the liquidity of the fund's investments and that offered to the fund's investor's, in the form of lock-up and redemption periods, are connected: hedge funds with longer lock-up and redemption periods may invest in more illiquid assets, which may provide greater profit opportunities.

The two aspects of hedge fund liquidity have very different properties. The illiquidity imposed on hedge fund investors by the fund itself, through the lock-up and redemption notice periods, has two important properties: it is constant through time and is directly observable, both to the investor (of course) and to the researcher. The illiquidity caused by the fund's investments in illiquid assets, on the other hand, will generally not possess either of these properties: the degree of liquidity will not necessarily be constant, and the degree of liquidity is not directly observable.

Using the first type of liquidity as a risk measure, Aragon (2007) finds that liquidity helps explain expected hedge fund returns. He shows that there is a positive, concave relationship between a hedge fund's excess returns and its redemption notice period. He also documents that the difference in excess returns on lockup versus non-lockup funds is about 4% per annum in aggregate.

The study by Getmansky, Lo and Makarov (2004) (henceforth GLM) on possible sources

[3] Ackermann, McEnally, and Ravenscraft (1999), Agarwal and Naik (2000), Edwards and Caglayan (2001), Fung and Hsieh (1997, 2002) and Liang (1999, 2000, 2001) provide comprehensive studies of historical hedge fund performance. Another strand of literature focuses more on the risk and return characteristics in specific hedge fund strategies. For example, Mitchell and Pulvino (2001) study the "risk-arbitrage" strategy; Fung and Hsieh (2001) focus on the "trend following" strategy and Agarwal and Naik (2004) study a number of equity-oriented strategies.

of serial correlation in hedge fund returns helps shed light on the harder-to-measure illiquidity resulting from investments. GLM note that correlation between reported returns on hedge funds in consecutive months (i.e., the first-order serial correlation, or autocorrelation, of the reported returns) is much higher than is commonly found in liquid equity or bond returns. They systematically examine numerous possible sources of serial correlation in hedge fund returns, and show that market inefficiencies, time-varying expected returns, time-varying leverage, and incentive fees with high water marks can all generate serial correlation in observed fund returns. However none of these sources can generate the magnitude of serial correlation found in the data. They conclude that the observed serial correlation must be the result of exposure to illiquid assets, which can lead to non-synchronicity problems in valuing the assets in the portfolio, inadvertent smoothing due to "marking to model" (as opposed to marking to market), and/or deliberate performance smoothing by managers[4]. Nonsynchronous trading of assets in a portfolio is well known to lead to autocorrelation in portfolio returns, see Shanken (1987) or Lo and MacKinlay (1990) for example. Marking to model will usually lead to serially correlated returns, even for sophisticated pricing models[5]. Further, by influencing *how* illiquid assets are marked to model hedge fund managers also have the ability to "manage" their reported monthly returns. Lo (2001) argued that given the nature of hedge fund compensation contracts and performance statistics, managers have an incentive to "smooth" their returns by marking their portfolios to less than their actual value in months with large positive returns so as to create a "cushion" for those months with lower returns[6]. This directly results in lower volatility of reported returns and higher Sharpe ratios, see Liew and French (2005). Agarwal *et al.* (2006) find evidence consistent with hedge fund managers manipulating their reported December returns so as to favourably

[4]The connection between (il)liquidity and serial correlation in equity returns has been explored by Campbell, *et al.* (1993) and Avramov, *et al.* (2005) for example.

[5]As GLM note, this is related to the fact that the conditional mean of a random variable is always smoother than the random variable itself.

[6]A number of studies have examined the impact of serial correlation in returns on estimates of performance or risk exposures, see Dimson (1979), Geltner (1991, 1993), Asness, *et al.* (2001), Brooks and Kat (2001), Okunev and White (2002), Conner (2003) and Liew and French (2005). Earnings smoothing has also been extensively documented in the studies of accounting management, see Beidlerman (1973), Bannister and Newman (1996), Subramanyam (1996). See also Chandar and Bricker (2002) and Goel and Thakor (2003).

influence their performance bonuses.

Building on the work of GLM, we exploit the link between autocorrelation in reported returns and exposure to illiquid assets to gain an insight into the time-varying liquidity of hedge fund returns by analyzing the time-varying nature of autocorrelation in hedge fund returns. Our analysis adds a time series dimension to the cross-sectional focus of GLM. GLM measure the illiquidity exposure as averages over the evaluation period, and these averages are taken "unconditionally" without regard to dynamic strategies hedge fund managers usually employ. Unlike traditional fund managers, the strategies of hedge funds are known to be dynamic, with fast turnover, involving both long and short positions, thus hedge funds usually have time-varying exposures to sources of risk. It is thus quite possible that the degree of illiquidity of hedge fund returns is also time-varying. By testing whether the degree of serial correlation (or liquidity) varies with certain variables we are able to draw some inferences as to the sources of this illiquidity. For example, how does the degree of illiquidity change with market returns or volatility, or with market-wide liquidity measures? By testing different candidates as the determinants of liquidity, we are able to produce a more complete picture of the sources of hedge fund (il)liquidity. Although the investments and strategies of individual hedge funds are closely-guarded secrets, we are able to indirectly capture the time-varying liquidity of their investments with our approach.

In a related paper, Bollen and Pool (2006) propose a method to distinguish between exposure to illiquid assets and intentional performance smoothing. Their approach is also based on allowing the serial correlation in hedge fund returns to vary through time. Given their tight focus on detecting fraudulent return smoothing, these authors only consider a single factor, namely whether the underlying "true" return on the fund is above or below some benchmark value. The inspiration for this factor stems from the insight that an unscrupulous hedge fund manager may report satisfactory returns more readily than losses, which generates time-varying serial correlation in reported returns. Our specification, described in Section 2, generalises that of Bollen and Pool to consider a wider variety of factors for time-varying liquidity in hedge fund returns. We also consider all funds in a style category jointly, which greatly increases the power of our method to detect significant liquidity factors.

We follow GLM and assume that reported hedge fund returns are some weighted average of the current and past q lags of true returns on the fund. We allow for time variation in liquidity by allowing the weights on current and lagged true returns to vary over time: a greater weight on the current true return means that more of the true return is reflected in the reported return, consistent with greater liquidity. Conversely, a lower weight on the current true return implies lower liquidity and higher smoothing. The weights are specified as simple functions of a collection of natural candidate variables for a study of liquidity: stock market returns, volatility and liquidity (as proxied by the factor of Pástor and Stambaugh, 2003), bond market returns, volatility and liquidity (as proxied by bid-ask spreads on Treasury bonds), as well as other factors such as a calendar dummy variable and a variable that tracks net flows into the fund. We estimate our model using monthly hedge fund returns from 1993 to 2004 from the CISDM database, and we find significant evidence of time variation in the liquidity of hedge fund returns for seven of the eight hedge fund styles considered. This is true whether we use a model with factors that are measured contemporaneously to fund returns, or a model where the factors are all lagged by one month. Somewhat surprisingly, our model using lagged factors, which may be used to predict future hedge fund liquidity, performs approximately as well as the model using contemporaneous factors. In a set of robustness checks, we show that our results hold for both live and "dead" funds, for funds that are open to new money or closed, and for audited versus non-audited funds.

For equity-based hedge fund strategies such as equity market neutral, equity hedge and non-hedge, liquidity decreases when stock market returns are low and bond market returns are high. For hedge fund strategies involving fixed income strategies, such as convertible arbitrage and fixed income arbitrage, liquidity decreases when equity market volatility is high. Fund flow is also a significant determinant of liquidity of these hedge fund strategies, but the direction of the impact of fund flow on hedge fund liquidity varies across strategies. For merger arbitrage and distressed securities, smoothing also increases during the middle of the calendar year. Overall, while we do not find evidence suggesting that the degree of serial correlation in hedge fund returns is directly linked to deliberate performance smoothing for most hedge fund strategies, we do find that for fixed income arbitrage funds, smoothing increases when the "true" return is negative. These results significantly extend the existing literature, and help us to think about the sources of hedge fund (il)liquidity.

The rest of the paper is organized as follows. In Section 2 we discuss our econometric model of time-varying liquidity. We present our set of potential liquidity factors in Section 3, and present our main empirical results in Section 4. In Section 5 we present the results of a simulation study of the finite-sample performance of our model for a variety of scenarios, and show that this performance is satisfactory for sample sizes relevant to our application. Section 6 concludes.

2 An econometric model of time-varying liquidity

As discussed in the Introduction, we implement our model of time-varying liquidity in hedge fund returns by exploiting a result of GLM: that hedge fund liquidity is inversely related to the degree of serial correlation in reported fund returns. To quantify the serial correlation in hedge fund returns, GLM assume that the fund i's reported return, denoted r_{it}^{o}, is a weighted average of its unobserved true returns, denoted r_{it}, over the most recent $q+1$ periods:

$$\begin{aligned} r_{it}^{o} &= \theta_{i0} r_{it} + \theta_{i1} r_{it-1} + \ldots + \theta_{iq} r_{it-q} \\ \text{where } 1 &= \theta_{i0} + \theta_{i1} + \ldots + \theta_{iq} \end{aligned} \tag{1}$$

Thus observed returns follow an MA(q) process. The constraint that the weights sum to one implies that the information driving the fund's performance in period t will eventually be fully reflected in observed returns, and this process could take up to $q+1$ periods from the time the information is generated. The parameter θ_0 can be regarded as a proxy for the illiquidity exposure of hedge funds: lower values of θ_0 imply that less of the current true return is reflected in the current reported return.

GLM estimate the above model assuming that the weights (i.e. the parameters of the MA process) are constant, and their results may be interpreted as a measure of a hedge fund's *average* illiquidity over the estimation period. To capture possible time variation of liquidity in hedge fund returns we propose allowing the parameters of the above model $(\theta_0, ..., \theta_q)$ to vary over time:

$$r_{it}^o = \theta_{i0t} r_{it} + \theta_{i1t} r_{it-1} + ... + \theta_{iqt} r_{it-q} \tag{2}$$

We assume that true returns are serially uncorrelated, in order to make the model well-identified, but we do not need to make any assumptions about cross-sectional correlation between the true returns on different funds, and we do not need to assume normality or homoskedasticity. We estimate our model via quasi-maximum likelihood.

In order to make the model empirically feasible given the limited histories available for most hedge funds, we are compelled to make the model as parsimonious as possible. As a step in that direction, we constrain the coefficients on lagged true returns to be a function of a single liquidity "index" variable, denoted δ_{it}, which completely determines the degree of liquidity of fund i at time t. The weights on the true returns are then constrained to decline geometrically towards zero, in a similar fashion to an autoregressive process[7]:

$$\begin{aligned} \theta_{ijt} &= \theta_{i0t} \cdot sgn(\delta_{it}) \left|\delta_{it}\right|^j, \quad j = 1, 2, \ldots, q \\ \theta_{i0t} &= \frac{1}{\bar{\theta}_{it}} \\ \text{where } \bar{\theta}_{it} &= 1 + sgn(\delta_{it}) \sum_{j=1}^{q} \left|\delta_{it}\right|^j \end{aligned} \tag{3}$$

where $sgn(x) = x/|x|$ for $x \neq 0$, and 0 for $x = 0$. By imposing geometric decay on the weights we effectively reduce the number of time-varying parameters to be modelled from q to just one, δ_{it}.[8] Following GLM we set $q = 2$, so each reported return is a weighted average of the three most recent true returns. To see how δ affects the weights on lagged

[7]We could, of course, parametrize the model as an autoregressive (AR) process rather than a moving average (MA) process. The main drawback of an AR specification is that it implies that any given true return takes an infinitely long time to be fully reflected in the reported returns, a feature that is at odds with the fact that most hedge funds are audited every twelve months. For this reason, we follow GLM and use a MA specification.

[8]We also estimated a fully-flexible MA(q) model on a sub-set of our funds, imposing no time variation in liquidity, and tested whether the geometric decay restriction was binding. The proportion of funds that rejected this restriction was in line with the size of the test, 5%, and so we concluded that this restriction is reasonable for our data.

true returns see Figure 1: With $\delta = 0$ there is no smoothing: the contemporaneous true return gets weight equal to one, and all lagged returns get weights exactly equal to zero. For $\delta > 0$ the weight on the contemporaneous true return decreases, and the weights on lagged returns increase. For $\delta = 0.5$ we see that the weight applied to the current true return is only about 0.6, indicating a substantial amount of smoothing. When $\delta < 0$ the weight applied to the contemporaneous true return is greater than one, and lagged returns have negative weights. This case corresponds to return 'inflation', whereby the current return is over-reported, and lagged returns are under-reported.

We specify the liquidity "index" variable, δ_{it}, for fund i as a simple function of common liquidity factors, denoted X_t, and fund-specific liquidity factors, denoted Z_{it}. Examples of the former include variables like stock or bond market volatility, while examples of the latter include net flows into fund i. We constrain δ_{it} to lie inside $(-1, 1)$ via the use of the modified logistic function, $\Lambda : \mathbb{R} \rightarrow (-1, 1)$

$$\delta_{it} = \Lambda(X_t'\lambda_i + Z_{it}'\phi_i) \quad (4)$$

$$X_t'\lambda_i = \beta_i + \gamma_{i1}X_{1t} + ...\gamma_{iM}X_{Mt} \quad (5)$$

$$Z_{it}'\phi_i = \phi_{i1}Z_{1t} + ... + \phi_{ip}Z_{pt} \quad (6)$$

$$\Lambda(z) = \frac{1 - e^{-z}}{1 + e^{-z}} \quad (7)$$

The model of GLM is obtained as a special case of our flexible model when $\gamma_{i1} = ... = \gamma_{iM} = \phi_{i1} = ... = \phi_{ip} = 0$. The model above has a total of $1 + M + p$ parameters per hedge fund (not including the unconditional mean and variance parameters). Our initial model has 7 common liquidity factors and 2 fund-specific factors, implying 10 parameters per fund. With the limited time series available on each fund (the median sample size is 46 months), this model is too heavily parameterised. One restriction that greatly enhances the information available for each parameter estimated is to impose that the liquidity factor coefficients are common across funds in a given style category. It would not be reasonable to assume that these parameters are constant across all the funds in our data set, but for funds within a given style this restriction is plausible. Thus we restrict $\gamma_{ik} = \bar{\gamma}_k \ \forall \ i$ and $\phi_{il} = \bar{\phi}_l \ \forall \ i$ for all funds (i) in the same style category, for each factor (k, l). This model thus has "intercepts", β_i, that vary across funds but factor coefficients that are constant across funds in the same style category.

All but one of our liquidity factors are variables that are directly measurable from the data, such as aggregate market returns or net inflows to the fund. Our last factor is specified as the sign of the "true" return on the fund. This factor relies on having an estimate of the true return, which is itself a product of the estimation. Analogous to the estimation of the efficient weight matrix for GMM estimation, we overcome this problem by iteration: We first estimate the model with all desired factors except the sign of the true return. From that model we obtain the estimated true returns, and thus the signs of the estimated true returns. We use this variable as an extra factor in a second estimation of the model. From the second estimation we again obtain the estimated true returns, and we use those in the third estimation, repeating the procedure until the parameters converge. Convergence was usually obtained in three to five iterations[9]. In our "predictive" model for hedge fund liquidity, based on lagged factors, we do not include the sign of the "true" return as a factor, and so no iteration is needed in estimation.

Our use of the sign of the "true return" as a liquidity factor is similar to the Bollen and Pool (2006) model for return smoothing. Their model was specified as a regime-switching AR(1) process, where the AR(1) coefficient varied according to whether the factor model assumed to be generating the hedge fund returns was above or below some threshold. Our specification is clearly similar in spirit to the Bollen and Pool specification, though we feel that our approach has some key advantages. Firstly, we allow for a wide variety of factors to affect the liquidity of hedge fund returns. This is important as, amongst other reasons, it allows us to control for well-known sources of time-varying liquidity, such as time-varying aggregate market liquidity, before drawing inferences about other possible liquidity factors. Secondly, by focussing just on the sign of the true return, rather than the sign of the fund

[9]We use the sign of the true return, rather than the level of the true return, as the iterative procedure described above did not always converge when the true return itself was used as a factor. When the sign of the true return was used we did not find any problems attaining convergence.

return in excess of some benchmark value, we avoid having to make any assumptions about the relevant factors for hedge fund returns (though of course we must do so for the factors affecting hedge fund *liquidity*). This is useful as there is no general consensus in the literature on an appropriate benchmark model for hedge fund returns. Finally, by imposing that the liquidity parameters are the same for all funds in a given style category our estimation procedure dramatically increases our power to detect significant liquidity factors. For seven out of eight styles considered we find evidence of time-varying liquidity that is significant at the 5% level (discussed in detail in Section 4). Bollen and Pool, on the other hand, detect significant time-varying serial correlation for only about 4% of their sample, which is approximately equal to the size of their tests (5%). This is likely due to their treatment of each fund separately, rather than pooling these funds by style category and gaining more precise estimates from cross-sectional data.

3 Description of the data and the liquidity factors

3.1 The determinants of time-varying liquidity

As determinants of time-varying hedge fund liquidity, we consider variables that are ex-ante likely to be related to aggregate market liquidity, such as market return, market volatility. A number of studies (see Pástor and Stambaugh, 2003 and Acharya and Pedersen, 2005 for example) have documented that aggregate liquidity does change over time and liquidity tends to drop when the market is down and volatility is high. In liquid markets, "marking to market" will generally be possible, and the degree of serial correlation in hedge fund returns will be low. When hedge funds' investments are illiquid, market prices are often not available and "fair values" will be determined by using a valuation model ("marking to model"). The calculated "fair value" is a prediction rather than an observation, which induces smoothing even if done as accurately as possible. Moreover, as GLM suggest, intentional performance smoothing is easier when investments are illiquid because of the subjective decisions that have to be made when estimating the "fair value" of an investment. Hence, we also consider variables which are directly related to intentional performance smoothing, such as the time until the next fund audit for example.

Market return: Asness *et al.* (2001) examine the relationship between hedge fund returns and lagged market returns separately for up and down markets. They find that coefficients on lagged negative market returns are larger than coefficients on lagged positive market returns, suggesting that more smoothing occurs when market returns are low. This could mean that fund managers deliberately smooth returns during market downturns, to reduce the reported losses, but do not smooth during market upturns. On the other hand, the results of Asness et al. may be a consequence of their choice of a single factor: hedge fund returns often display option-like properties, as shown by Fung and Hsieh (1997) for example, and apparent asymmetric factor exposures can occur when asset returns are more or less option-like than the chosen factors, see Jagannathan and Korajczyk (1986). Another explanation, unrelated to intentional smoothing, is that aggregate market liquidity generally decreases during market downturns[10], making it harder for managers to mark-to-market, thus inducing greater smoothing in downturns than upturns. We will include proxies directly aimed at controlling for aggregate market liquidity, see below, and thus we isolate the impact of market returns on hedge fund liquidity separately from aggregate market liquidity.

We use the return on the S&P 500 index as the proxy for equity market returns and the return of Lehman Brothers aggregate bond index return as the proxy for bond market returns. The Lehman Brothers index covers government securities, mortgage-backed securities, asset-backed securities and corporate securities to simulate the universe of bonds in the market. The maturity of the bonds in the index are all greater than one year.

Market volatility: The well-known positive relation between volatility and volume suggests that in more volatile markets we expect to see more assets being marked to market, rather than marked to model, and thus less smoothing. However, when volatility is high, liquidity is usually low, see Pástor and Stambaugh (2003) and Chordia, *et al.* (2005) for example. Therefore, high market volatility may lead to less market-wide liquidity, and thus

[10]See Chordia, *et al.* (2005) for example. Brunnermeier and Pedersen (2006) provide one theoretical explanation as to why liquidity may "dry up" more often when markets decline.

more smoothing in hedge fund returns. As proxies for market volatility, we follow French, Schwert and Stambaugh (1987) and estimate monthly stock and bond market volatility as

$$\sigma_t^2 = \sum_{i=1}^{N_t} r_{it}^2 + 2\sum_{i=1}^{N_t-1} r_{it} r_{i+1,t} \tag{8}$$

where r_{it} is the daily return on the S&P 500 or Lehman Brothers bond index on day i in month t, and N_t is the number of trading days in month t.

Market-wide liquidity: The markets in which hedge funds trade also go through periods of high and low liquidity (see footnote 2 for references). Fluctuations in aggregate market liquidity will almost mechanically induce time-varying liquidity in hedge fund returns. Further, periods of low market liquidity correspond to periods when managers can exercise greater discretion in marking-to-model, thus perhaps inducing even larger fluctuations in liquidity than in the underlying markets. Both of these observations imply that it is important to control for market-wide liquidity before attempting to infer a relation between hedge fund liquidity and other factors.

Our stock market liquidity measure is the Pástor and Stambaugh (2003) liquidity index from CRSP database. Their monthly measure of aggregate stock market liquidity is a cross-sectional average of individual stock liquidity measures estimated using daily data, which is based on the principle that lower liquidity corresponds to greater volume-related return reversals. The rationale behind this measure is consistent with the model and empirical evidence presented by Campbell, *et al.* (1993); see also Avramov, *et al.* (2005). These authors find that returns accompanied by high volume tend to be reversed more strongly, and they explain how this result is consistent with a model in which some investors are compensated for accommodating the liquidity demands of others.

Fleming (2003) reports that the quoted spread is the best measure for monitoring Treasury bond liquidity, and Goldreich, *et al.* (2005) suggest that quoted spreads are a better measure of liquidity risk in bond market than effective spreads. We follow these papers estimate bond market liquidity using quoted bid and ask prices available from the CRSP Treasury Quotes file. This measure is also used by Goyenko (2005).

Fund flows: Fund flows directly affect the hedge fund managers' income because managerial fees are dependent on the size of the fund. Flows, of course, also impact the hedge fund manager's ability to take on positions: capital out-flows force the fund manager to liquidate his positions, thus leading to more marking to market and less smoothing of returns. On the other hand, if a fund manager expects more fund outflows following poor past performance, then in order to reduce outflows he may be more inclined to smooth reported returns.

We compute the percentage net fund flow in month t as the difference between the growth in the total net asset value of the fund and the reported return on the fund. In our first model, we use a forward-looking three month moving average of fund flow as our proxy for fund flows since the date of an out-flow will be known in advance due to redemption notification rules, whereas an in-flow to a fund may or may not be anticipated. In our "predictive" model for hedge fund liquidity we simply use the net fund flow for the previous month.

The sign of "true" return: When "true" returns are positive, part of the gains/losses earned in a given month may not be reported so as to off-set future losses/gains, thus smoothing increases. The SEC (2004) emphasized that valuation problems often arise "when hedge fund advisers overstate assets in order to cover trading losses." On the other hand, it is also possible that low/negative returns on the fund lead to more redemptions, which induces unwinding of positions, enabling marking-to-market, which improves liquidity.

The time until the next fund audit/calendar effects: If hedge fund auditors provide independent confirmation of annual returns, then any intentional smoothing that takes place during the financial year must be "unwound" by the year-end. If hedge fund managers were intentionally smoothing their reported returns, then it would be easiest to do so in the months furthest from an audit date. Unfortunately, the dates of fund audits is not generally publicly available, although Liang (2003) reports that hedge funds are usually audited in December, and Agarwal, et al. (2006) note that bonuses are paid in December[11].

[11] Herzberg and Mozes (2003) and Agarwal, et al. (2006) find that hedge fund returns, as opposed to serial correlation in these returns, are not equally distributed across calendar months: on average hedge funds report returns in December that are much larger than other months during the year. Agarwal, et al. (2006)

Thus our measure of the time until the next fund audit is a general "calendar effect" variable, and any seasonal pattern in hedge fund liquidity could also come from a seasonal pattern in aggregate market liquidity. A significant such pattern in U.S. equities is reported by Chordia, *et al.* (2005) and Hong and Yu (2005). To capture seasonal pattern in hedge fund liquidity, we define the time dummy equal to zero from April to September, one in the rest of the months of the year.

3.2 Description of the hedge fund returns data

We use monthly returns and accompanying information on both live and "dead" individual hedge funds from January 1993 to August 2004 from the CISDM database. This database provides monthly observations of returns, total net assets, and net asset values, and a fund information file, containing fund name, strategy type, management fees, and other supplementary details. We analyze 8 fund strategies, namely Merger Arbitrage, Distressed Securities, Equity Hedge, Equity Nonhedge, Market Neutral, Fixed Income Arbitrage, Convertible Arbitrage, Global Macro. Due to the data-intensive nature of the research question, we only study funds with at least 48 months of observations, which leaves us with a total of 609 individual hedge funds.

In Table 1 we provide summary statistics on the hedge fund returns over our sample period. For each strategy, the table lists the number of funds and means and standard deviations of basic summary statistics and the smoothing profiles estimated using the GLM MA(2) smoothing process. Consistent with GLM, merger arbitrage, distressed securities, convertible arbitrage and fixed income have relatively high average serial correlations and the estimated smoothing parameter $\hat{\theta}_0$ is low on average. Market neutral, equity hedge, equity nonhedge, global macro funds which mainly invest in liquid assets exhibit low serial correlation and $\hat{\theta}_0$ is relatively close to unity. Panel A in Table 2 contains summary statistics of our liquidity factors over the sample period. Panel B in Table 2 reports pairwise correlation among the various determinants, and shows that none have excessively high

argue that this is because fund managers engage in managing their returns towards the end of the year in order to earn greater incentive fees.

correlation.

4 Empirical evidence of time-varying hedge fund liquidity

We present our results in two formats. The first set of results, in Table 3, includes all of the factors discussed in the previous section. The results in this table allow us to examine the impact of a given factor, *controlling for the influence* of other factors that are possibly relevant for hedge fund liquidity. The cost of including these extra variables is the generally decreased significance of any given individual factor. However, the fact that our model controls for the influence of other relevant variables is an important advantage of our approach.

In Table 4 we report the model resulting from a general-to-simple specification search, where we started with the most general model, and then dropped the factor with the lowest t-statistic and re-estimated the reduced model. We continued eliminating variables, one at a time, until all the factors left in the model were significant at 10% level[12]. By excluding variables which are not statistically significant we parsimoniously extract factors that are significant determinants the liquidity in hedge fund returns[13]. All standard errors reported in this paper are Newey-West (1987) robust standard errors.

Our initial model imposed that the coefficients on the liquidity factors were common across funds in the same style category, but allowed the "intercepts" (β_i in equation 5), or unconditional levels of serial correlation, to vary across each fund in a given style. We tested the additional restriction that the intercept parameters were equal across funds in the same style category and found that this restriction was not rejected for any of the styles we consider. Thus the results we report are from the simplified model with the intercepts imposed to be constant across funds in the same style category.

[12]Some detailed discussion of the general to simple model selection procedure can be found in Hendry and Richard (1982, 1983) and Hoover and Perez (1999).

[13]Interestingly, the parameter estimates and t-statistics in the reduced model are not very different from those in the most general model except for equity nonhedge funds, where some insignificant factors in the general model turn out to be significant in the reduced model.

Before discussing the results for particular hedge fund styles, let us firstly describe the results in general. Tables 3 and 4 reveal quite clearly that stock market returns and volatility are important determinants of hedge fund liquidity. The influence of these factors is not merely due to them proxying for market liquidity, since the models reported in Table 3 include controls for stock and bond market liquidity. For five out of the eight styles considered we find stock market returns or volatility to be significant determinants of hedge fund liquidity. The return on the bond index is also an important variable, being significant for three styles. Somewhat surprisingly, the stock market liquidity proxy is not significant for any style (in the general model; it is significant in one of the reduced models) and the bond market liquidity proxy is only significant for one style. This suggests that while these proxies track liquidity in the stock or bond markets well, they are not as good at tracking time-varying liquidity in hedge fund returns. The only remaining variable that was often significant across styles is the fund flows variable: this variable was significant in four out of the eight styles. Interestingly, the sign of this variable was not consistent across styles: for Merger and Convertible Arbitrage styles the sign was negative, indicating that net in-flows lead to higher liquidity, whereas for the Fixed Income and Global Macro styles the sign was positive, indicating that net *out*-flows lead to higher liquidity.

To glean some further insight from our models, in Figures 2 and 3 we plot the time series of $\hat{\theta}_{0t}$, estimated from the general model, for each hedge fund style. For comparison purposes, we also plot the GLM estimate of $\hat{\theta}_0$ for each style and a 95% confidence interval for this estimate. These figures clearly show that the liquidity of hedge fund investments varies significantly over time across all fund styles. Consistent with conventional wisdom that the assets of equity-based funds are more liquid, the general level of $\hat{\theta}_{0t}$ of equity-based styles is nearer to unity than those of non-equity based styles. However, by allowing θ_0 to vary through time important variations in liquidity are found even for funds that have an average θ_0 (the "GLM θ_0") that is near unity. For example, the Equity non-hedge style both has an average θ_0 very near one, whereas the time-varying estimates show that θ_0 for this styles varies from as low as 0.5 (indicating substantial return smoothing) up to as high as 2.5 (indicating return "inflation").

Interestingly, our model detects a large drop in liquidity around September 1998 in all fund styles except Global macro. This period corresponds to the time of the Russian debt crisis and the collapse of LTCM, a period widely-thought to have been one of low aggregate liquidity, see Pástor and Stambaugh (2003) and Brunnermeier and Pedersen (2006) for example. Another large downward spike in our measure of liquidity occurs around July 2002, particularly for the Merger arbitrage, Distressed securities and Convertible arbitrage styles. This corresponds roughly to the outbreak of now-infamous accounting scandals (Enron, Arthur Andersen, Adelphia, and WorldCom). In the plots of Merger arbitrage and Distressed securities funds, time-varying $\hat{\theta}_{0t}$ also exhibits a strong calendar effect, with liquidity falling in the middle of the calendar year and rising around the end of the year.

4.1 Results for individual hedge fund styles

Determinants of liquidity in market neutral fund returns: Managers of market neutral funds purchase undervalued securities and short sell overvalued securities so as to neutralize the impact of the overall market. This strategy is regarded as a "zero beta" strategy because the whole portfolio is supposed to have no, or at least low, co-movement with the overall market, although Patton (2005) reports some evidence against this. We find just two significant determinants of liquidity in market neutral fund returns: stock market returns and bond market returns. These factors are significant in the general model and are the only two factors that appear in the reduced model. The general model is significant overall, with a joint test of the hypothesis that all factors have coefficients equal to zero being rejected with a p-value of 0.02. We find that stock market returns are positively related to market neutral hedge fund liquidity even after controlling for market liquidity. This finding is consistent with the results of Asness et al. (2001), who find that coefficients on lagged negative market returns are larger than coefficients on lagged positive market returns, implying greater smoothing during market downturns. This may be due to managers' intentional performance smoothing during market downturns. However we do not find the sign of "true" return is a significant determinant of the degree of liquidity, which would seem a more relevant indicator of intentional performance smoothing. Further, we find that bond returns have a negative influence on hedge fund liquidity, in that high bond market returns lead to lower liquidity, which is hard to reconcile with an intentional performance smoothing explanation.

Determinants of liquidity in equity hedge fund returns: Like equity market neutral funds, equity hedge fund investments also combine holdings of long equities with short sales of stocks and/or stock index options. The difference from market neutral funds is equity hedge fund managers do not aim for a zero beta. They usually increase net long exposures in bull markets and decrease net long exposures or even are net short in bear markets. Similar to equity market neutral funds, we find a positive (negative) relation between stock (bond) market return and the degree of liquidity in equity hedge fund returns. The joint significance of time variation in liquidity is again strong, with a p-value of less than 0.01.

Determinants of liquidity in equity nonhedge fund returns: Equity nonhedge funds are commonly known as "stock-pickers" because these funds are predominately long in equities. The important distinction between equity non-hedge funds and equity hedge funds is equity non-hedge funds do not always have a hedge in place. Similar to equity hedge funds and market neutral, we find that stock market returns are positively related to the liquidity in equity nonhedge fund returns. However, instead of bond return as a significant determinant of liquidity, we find there is a negative relationship between stock market liquidity and the liquidity in equity nonhedge fund returns. This is the only style category in which the aggregate stock market liquidity proxy was significant, and it is only significant in the reduced model; in the aggregate model the coefficient on this variable has a t-statistic of only 1.21. Overall, however, the general model for time-varying liquidity is very significant, with a p-value from a joint test of constant liquidity being less than 0.01.

Determinants of liquidity in global macro fund returns: Global macro funds aim to profit from changes in macroeconomies, and participate in equities, bonds, currencies and commodities markets. For this style, we found bond market returns to be positively related to the degree of liquidity in global macro fund returns, indicating lower liquidity for these funds when the bond market is falling. The only other significant factor for this style was the fund flow. This factor has a positive coefficient, indicating that net *out*-flows (meaning the "net fund flow" variable takes a negative value) *increase* the liquidity of the fund. This finding is consistent with the scenario where out-flows cause the fund to liquidate some of their positions, which enables marking to market and thus greater liquidity. The global macro style is the only category where the general model for time-varying liquidity could not reject the null of constant liquidity; the p-value for this test is 0.15.

Determinants of liquidity in merger arbitrage fund returns: Contrary to previous four strategies, the merger arbitrage strategy usually involves less liquid assets. Merger arbitrage funds take bets in event-driven situations such as mergers, takeovers, corporate restructuring, spin-offs. In the general model we find only one significant factor: net fund flows. The sign of this coefficient is negative, indicating that net out-flows decrease liquidity. This finding is not consistent with the scenario where out-flows cause the fund to liquidate some of their positions, enabling marking to market. Instead, the sign of this coefficient is consistent with the scenario where a hedge fund manager experiencing net fund out-flows increases the degree of intentional performance smoothing, thus decreasing our estimate of liquidity[14].

Determinants of liquidity in distressed securities fund returns: This type of fund usually focusses on companies which have been, or are expected to be, in financial difficulties. Investments may be made in bonds, stocks, bank debt, corporate debt, trade claims and/or warrants. Distressed securities are often illiquid and it is not always possible to mark them to market. In contrast to our results for market neutral and equity hedge funds, we find that stock market volatility shows a significant negative relation with liquidity, suggesting that when the stock market is volatile the liquidity of these funds is lower. We also find bond

[14]In the reduced model we also find that the calendar dummy is significant and positive, indicating that liquidity is significantly lower in the April-September period than the October-March period. This is consistent with the hypothesis that it is easier for hedge fund managers to intentionally "smooth" their reported returns in the months furthest from an audit date, i.e., in the middle of the calendar year. However it is important to note that this variable is not significant in the general model, where we control for many other relevant variables. If aggregate market liquidity is also seasonal, as found by Chordia, *et al.* (2005) and Hong and Yu (2005), then by omitting it as a control variable we may spuriously find seasonality in hedge fund liquidity.

market return is positively related to distressed securities fund liquidity, though only in the reduced model.

Determinants of liquidity in convertible arbitrage fund returns: Convertible arbitrage strategies attempt to take advantage of relative pricing discrepancies between the theoretical and market prices of convertible bonds. If a convertible bond appears to be undervalued, then the manager may take a long position in the convertible bond and short the company's equity to reduce the exposure to equity risk. For these funds we again find that higher stock market volatility leads to lower liquidity of these funds' investments. We also find our aggregate bond market liquidity proxy to be significant, and of the expected sign: lower bond market liquidity leads to lower hedge fund liquidity. Finally, we also found that the net fund flows are a significant factor for these funds. The sign of the coefficient is negative, again suggesting that net fund out-flows decrease the liquidity of the fund's investments.

Determinants of liquidity in fixed income fund returns: This strategy attempts to exploit mis-pricing among fixed income securities while neutralizing exposure to interest rate risk. The restriction of constant liquidity is most strongly rejected for this style of hedge funds: the test statistic is 53.16, compared with the critical value of 16.92 at the 0.05 level. As in convertible arbitrage funds, we find liquidity is lower when stock market volatility is high. Particular to fixed income arbitrage funds, the sign of "true" return shows a significant positive relation with liquidity. This is consistent with a scenario where the hedge fund manager intentionally smooths negative returns, while more honestly reports positive returns. It is this type of pattern in hedge fund returns that Bollen and Pool (2006) sought to detect as a potential indicator of fraudulent reporting.

Summary of the determinants of time-varying liquidity in hedge fund returns: Overall, we find substantial evidence of time variation in the amount of smoothing in hedge fund returns. For all but one of the eight style categories studied we were able to reject the null of constant liquidity at the 0.05 level. For equity-based hedge fund strategies, such as equity market neutral, equity hedge and non-hedge, which usually invest in liquid securities and exhibit low serial correlation in their returns, liquidity decreases when stock market returns are low and bond market returns are high. For strategies focussed more on fixed income markets, such as convertible arbitrage and fixed income hedge funds, liquidity decreases when equity market volatility is high. Fund flow is also a significant determinant of liquidity of these less liquid hedge fund strategies. Overall, we do not find evidence of deliberate performance smoothing for most hedge fund strategies, but we do find that for fixed income arbitrage funds, smoothing increases when the "true" return is negative.

4.2 Predicting hedge fund liquidity

Our results above strongly suggest that the liquidity of hedge fund investments is time-varying, and that this variation can be at least partially captured using contemporaneous values of such factors as equity market returns and volatility, bond market returns and net fund flows. A natural follow-up question to ask is whether we can *predict* hedge fund liquidity using these factors. We investigate this question by estimating the same model but using one-month lags of our predictor variables[15]. We drop the "true return" factor from this specification, and we use simply the one-month lag of net fund flows, rather than a (forward-looking) three-month average of net fund flows. The results of this specification are presented in Table 4.

The results from this predictive model are broadly consistent with the "contemporaneous" model discussed in the previous section. Seven out of eight styles again exhibit significant time variation in liquidity, although it is the Equity non-hedge style that has no significantly predictable liquidity, whereas in the contemporaneous model it was the Global macro style that appeared to have constant liquidity. For the remaining styles the predictability of fund liquidity is strongly significant: for the Market neutral style the p-value on the test for no predictability is 0.01, while for the remaining styles they are less than 0.001. Thus hedge fund liquidity appears to be both significantly time-varying, and significantly predictable.

Two main features of the results for the predictive model of hedge fund liquidity stand out. Firstly, the one-month lagged return on the S&P 500 index is a strongly significant

[15]This is in the spirit of the "direct" forecasting approach as opposed to the "iterated" approach, which was found by Marcellino, Stock and Watson (2006) to be preferable for forecasting macroeconomic variables.

factor for predicting hedge fund liquidity for seven out of the eight styles. In all cases, the sign of the coefficient on the S&P 500 return is negative, implying that falls in the stock market are followed by decreases in fund liquidity. This is the most consistent result of all the factors, including in the contemporaneous factors. The second prominent finding is for the Pástor-Stambaugh equity liquidity index, which is significant for three styles (Distressed securities, Convertible arbitrage, and Fixed income) and has in all three cases a negative sign, indicated that falls in aggregate liquidity predict falls in fund liquidity the following month.

4.3 Robustness checks

In this section we investigate the robustness of the above findings to hedge fund characteristics beyond stated investment styles. We consider sorting funds by whether they were still reporting to the database as at the end of our sample period or not ("live" versus "dead" funds), whether they were accepting new money at the end of our sample period or not ("open" or "closed"), and whether they had been audited by the end of our sample period.

"Live" versus "dead" funds: We consider this split of the funds to determine whether we can reasonably pool "dead" funds with live funds, the latter of course being of primary interest to investors and regulators, to gain more precise parameter estimators. There were 361 funds still reporting at the end of our sample period, leaving 248 "dead" funds. We estimated our model on each of these two groups of funds, and report the results in the first two columns of Table 6. For both of these groups of funds we find the bond market return to be the only significant liquidity factor. It has a positive coefficient, as it did for the market neutral, equity hedge and global macro fund styles, presented in Table 3, indicating that higher bond market returns are associated with lower hedge fund liquidity. Furthermore, the factor coefficient estimates for both of these groups of hedge funds are very similar. These results suggest that the time-varying properties of fund liquidity are not substantially different for live and dead funds.

"Open" versus "closed" funds: One might reasonably think that the incentives facing a hedge fund manager who is not accepting new money are different to those facing a manager who is still accepting new money into his/her fund. Most relevant to this study, the incentives to intentionally smooth returns are presumably greater for managers still looking to increase the size of their fund than those who have closed the fund to new money. There were 308 live funds still open to new money at the end of our sample period, 48 funds were closed to new money, while for five funds we had no information on whether they were still accepting new money or not. We excluded these five funds from this analysis. The estimation results are presented in the third and fourth columns of Table 6. For both of these groups of funds we again find the bond market return to be the only significant liquidity factor: it has a positive coefficient, indicating that higher bond market returns are associated with lower liquidity for both open and closed funds. We also again find that the coefficient estimates are very similar across these two groups of funds, and so we conclude that the liquidity properties of open and closed funds are not substantially different.

Non-audited funds: If the autocorrelation in reported hedge fund returns was primarily due to intentional return smoothing, rather than just a side-effect of investment illiquidity, then one may expect the model results to differ between funds that had been subject to an audit and those that had not. Almost all of the funds in our database were audited: only 20 out of 609 were not. Of these, 13 were live funds and 7 were dead funds. We estimated our model on the non-audited funds, but it was not feasible computationally to estimate the model on all 589 audited funds jointly. Thus we compare the results for the non-audited funds, presented in the final column of Table 6, with those for the live and dead funds presented in the first two columns. None of the factors for the non-audited funds were significant at the 5% or 10% level, however the model overall had a p-value of 0.06, and so is significant at the 10% level. This slightly reduced significance may be attributable to the fact that there are only 20 funds in this category, and so the amount of cross-sectional information available is limited. The individual coefficient estimates are comparable to those for live and dead funds as a whole, and thus we conclude that non-audited funds are not substantially different from the rest of the funds in our database, and need not be excluded from the pooled estimation of the model.

5 A simulation study

With the relatively short samples of hedge fund returns available, and the somewhat complicated nature of our model, it is important to verify that the properties of our estimator are satisfactory for realistic sample sizes. We do this via a small simulation study of our model for a range of realistic scenarios. We are primarily concerned with the finite-sample size of the test, that is, the proportion of times a true null hypothesis is rejected. The null hypotheses of interest in our paper relate to the individual and joint significance of factors for the time-varying liquidity of hedge fund investments.

With this in mind, we simulate from a simple model for hedge fund returns where the liquidity is constant, and then estimate our model which allows for time-varying hedge fund liquidity and test for the significance of the factors in the model. Ideally, the proportion of times that a factor is found to be significant should roughly equal the nominal size of the test, which will be set at 0.05 throughout. The data generating process for the hedge fund returns and the liquidity "factors" is:

$$\begin{aligned} r_{it}^{o} &= (1-\theta_{i1}-\theta_{i2})\, r_{it} + \theta_{i1} r_{i,t-1} + \theta_{i2} r_{i,t-2}, \quad t = 1,2,...,T; \ i = 1,2,...,K \\ \begin{bmatrix} r_{it} \\ \mathbf{f}_t \end{bmatrix} &\sim iid\ N\left(\begin{bmatrix} \mu_i \\ \mathbf{0} \end{bmatrix}, \begin{bmatrix} \sigma_i^2 & \mathbf{0}_N' \\ \mathbf{0}_N & I_N \end{bmatrix} \right) \end{aligned}$$

where I_N is an identity matrix of dimension N and $\mathbf{0}_N$ is a $(N \times 1)$ vector of zeros. Thus we assume that the true returns on hedge fund i, r_{it}, are $iid\ N\left(\mu_i, \sigma_i^2\right)$. The reported returns, r_{it}^{o}, are then generated as a linear combination of the three most recent true returns, following the model of GLM. We specified the values for $(\mu_i, \sigma_i^2, \theta_{i1}, \theta_{i2})$ from estimates of a MA(2) model on a randomly selected subset of our funds. The liquidity "factors" were set as simple *iid* standard Normal variables. As the factors in this simulation are *iid* and independent of the true return, the results of this simulation can be used to justify both specifications (contemporaneous and predictive) of our model.

An important empirical feature of our data is the unbalanced nature of our panel of hedge fund returns. By using both live and dead funds in our analysis we are forced to deal with the fact that some funds drop out of our sample before the last period, and many funds were not present in the first period. In order to replicate this feature of the data in our simulation we randomly selected fifty (the largest value of K we considered in the simulation) funds from our sample and recorded the dates of each fund's first and last observation, t_i^{first} and t_i^{last}. From these, we computed

$$\tau_i^{first} = \frac{t_i^{first}}{T}, \quad \tau_i^{last} = \frac{t_i^{last}}{T}$$

which reflect the proportions of each sample that were missing from the start and end of the sample for fund i. These are helpful when considering various values of T; if we instead just matched the values of $\left(t_i^{first}, t_i^{last}\right)$ then for large T the impact of missing data would go to zero. To replicate the missing data in our simulation we used these values of $\left(\tau_i^{first}, \tau_i^{last}\right)$, $i = 1, 2, ..., 50$ to determine which observations we should "throw away". In the tables below we report the results both with and without the missing observations imposed on the simulated data.

The key design parameters in this simulation are the sample size, T, the number of funds in the category, K, and the number of factors, N. Our application involved sample sizes of between 48 and 140, style categories containing between 20 and 121 funds, and a set of factors ranging from 2 to 8. With this in mind, we used the following: $T = 75,\ 150,\ 500$; $K = 1,\ 10,\ 50$; $N = 1,\ 4,\ 8$. Ideally we would have also ran the simulation for larger values of K but this was not computationally feasible, given that we need to replicate each scenario hundreds of times to get precise estimates of the finite-sample rejection frequencies. Even this small simulation study was quite computationally burdensome[16].

The largest value of T we consider is much larger than our longest time series, but

[16]This simulation took approximately 2520 hours (over 100 days) on a Pentium 2.8GHz machine, for 300 replications.

we include this value as a check on the quality of the asymptotic approximation: if the finite-sample rejection frequencies are substantially different from their nominal levels when $T = 500$ then this may indicate a problem with the assumptions underlying the asymptotic theory. Unsurprisingly, we find that the $T = 500$ scenario leads to satisfactory results in almost all cases.

We present the results for two scenarios: one where there were no missing observations and where the sign of the "true" return is not included as a factor, and a scenario where we imposed some missing observations (described above) and included the sign of the "true" return as a factor[17]. We considered the case with and without the "true return" factor as this factor requires an iterative estimation procedure, whereas the other factor coefficients can be estimated as usual via maximum likelihood. It turned out that the coefficient on the "true return" factor was well-estimated using our iterative procedure.

In Table 7 we present the proportion of times that a single factor was found to have a coefficient significant at the 0.05 level. Since the true coefficients on all factors in the simulation are zero, these proportions should be approximately equal to 0.05. This is generally what we find. The worst results are obtained when only a single fund is included in the panel ($K = 1$), and when the number of factors considered is large ($N = 4, 8$). Most importantly for our study, the results for the scenarios with $T = 75, 150$ with $K = 50$ and $N = 4, 8$ are close to the nominal size of 0.05. Thus we can be reasonably confident that our t-tests have satisfactory finite sample properties.

In Table 8 we present the proportion of times that we could reject the null hypothesis that *all* factors have coefficients equal to zero. These results are worse than the individual t-test results, particularly when $K = 1$ and $N = 4, 8$. However, the scenarios with $T = 75, 150$ with $K = 50$ and $N = 4, 8$, which are the most relevant for our study, are reasonably close to the nominal size of 0.05. The tendency for this joint test to slightly over-reject the null hypothesis suggests that we should be careful interpreting results where the rejection of the

[17] In unreported results (available from the authors upon request) we also considered the two other cases, involving missing observations but without the "true return" factor, or involving no missing observations and including the "true return" factor. The bulk of the differences between the two cases presented here is attributable to the creation of missing observations; the impact of including the "true return" factor is minimal.

null is "borderline". We rejected the null of constant liquidity for seven of our eight styles, and for six of the eight styles the p-value was less than 0.001. For the market neutral style the p-value was 0.022, and so these simulation result may indicate that the evidence against constant liquidity is slightly weaker for this style than the asymptotic theory would suggest.

Overall, we conclude from this simulation study that our econometric approach has reasonable properties in finite samples, particularly when the funds are "pooled" together and treated jointly. With longer samples ($T = 500$) our simulation results suggest that it is feasible to treat each fund separately, thus allowing greater flexibility in modelling time-varying liquidity, but for the sample sizes currently available ($T = 75, 150$) pooling of funds is a simple way of improving the accuracy of the tests.

6 Concluding remarks

In this paper we proposed a method to identify and analyze the key determinants of hedge fund liquidity. Our approach does not require information on the actual positions taken by the hedge fund, rather we use only the monthly returns reported by the hedge fund and other easily observed information. We find substantial evidence of time variation in the liquidity of hedge fund returns, and this variation can be predicted with readily available data. Hedge funds in equity-based styles, such as equity market neutral and equity hedge or non-hedge, exhibit decreases in liquidity when stock market returns are low and bond market returns are high. In contrast, hedge funds in fixed income styles, such as convertible arbitrage or fixed income arbitrage, exhibit lower liquidity when equity market volatility is high, and when the fund experiences in-flows or out-flows of funds.

Our methodology and empirical findings on the key determinants of hedge fund liquidity have important implications for investors in and regulators of hedge funds. It is usually very difficult for hedge fund investors to closely monitor liquidity risk of hedge funds, since the risk exposure of hedge funds can change dramatically in response to market conditions. Our methodology provides an alternative means for investors to capture time-varying liquidity of hedge fund investments. It is also useful for regulators to monitor potential market liquidity risk resulting from the opacity of hedge funds' investments. While direct regulation of hedge fund positions would require comprehensive (and sensitive) information, our approach offers a convenient (and less controversial), way to estimate the liquidity of hedge fund investments since only the reported returns on the fund and other currently available data.

The identification of key determinants of hedge fund liquidity also sheds some light on the issue of evaluation of hedge fund assets. The accurate and impartial valuation of hedge fund assets has become an increasingly important issue in the hedge fund industry[18]. Our results indicate that especially for some hedge fund strategies involving particularly illiquid assets, such as the fixed income style, a consistent valuation standard or an independent administrator appointed to carry out the valuation may be needed.

Whilst our results shed new light on the factors that are correlated with the liquidity of hedge fund investments, our reduced-form econometric model is not able to definitively separate the possible causes of these observed correlations. For example, equity market returns were found to be an important factor for several of the style categories we considered, and we found that periods of low equity returns correspond to periods of low hedge fund liquidity. One scenario that would generate such a correlation is if funds find it harder to mark to market in falling markets. In that case, falling markets correspond to times when funds rely more heavily on "marking to model", which is revealed as lower liquidity in our approach. Another, less benign, scenario is one where the incentives to intentionally smooth reported returns lead fund managers to indulge in this practice more often in falling markets. Separating these two scenarios requires a theoretical model with careful descriptions of the preferences and incentives faced by the relevant agents: managers, investors, and regulators. Our empirical results will be useful in checking the realism of the predictions from this type of model, but we leave the development such a model to future research.

[18]Whether the significant autocorrelation in hedge fund returns is due to hedge funds' systematic illiquidity exposure or managers' intentional performance smoothing is directly related to the question whether the valuations of hedge fund assets are accurate and impartial. The answer to this question is of particular importance to regulators and was the central topic of a speech given by the United Kingdom Financial Service Authority's asset management sector head (Dan Waters) at a conference in March 2006. He noted that "In 2005, valuation related losses in hedge funds were estimated to total \$1.6 billion. Poor valuation procedures in combination with weak internal controls were in some cases exploited to misrepresent hedge fund valuations and commit fraud."

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Table 1

Summary statistics for hedge fund returns

This table presents cross-sectional means and standard deviations of basic summary statistics for funds in the CISDM database over the sample period January 1993 to August 2004. K is the number of funds. SD denotes standard deviations. As in GLM, $\hat{\theta}_0$ is the estimated smoothing parameter of the MA(2) smoothing process $r_t^o = \theta_0 r_t + \theta_1 r_{t-1} + \theta_2 r_{t-2}$, where r_t^o is the observed return on the fund, subject to the normalization $1 = \theta_0 + \theta_1 + \theta_2$, and estimated via maximum likelihood. $\hat{\rho}_1\%$ and $\hat{\rho}_2\%$ denote first order and second order autocorrelation respectively.

Category	K	Mean		SD		Skewness		Kurtosis		$\hat{\theta}_0$		$\hat{\rho}_1\%$		$\hat{\rho}_2\%$	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Mkt neutral	121	0.96	0.57	3.88	2.98	0.44	1.19	6.57	5.70	0.88	0.24	13.85	17.72	7.54	15.27
Eq hedge	58	0.99	0.84	5.18	2.74	0.01	1.10	6.40	4.85	0.92	0.37	13.99	16.23	6.88	14.16
Eq nonhedge	20	1.24	0.82	8.25	4.69	0.17	0.69	4.86	1.88	0.97	0.18	6.97	13.16	-0.01	11.30
Global macro	90	0.96	0.81	5.30	3.54	0.28	0.97	5.60	4.40	0.97	0.27	9.51	16.81	1.98	15.14
Distressed	72	1.08	0.59	3.82	3.01	-0.14	1.34	7.83	6.36	0.84	0.31	18.63	16.85	7.81	13.63
Merger arb	106	0.89	0.54	3.04	3.72	-0.17	1.14	6.70	5.20	0.82	0.15	20.67	15.82	11.63	15.57
Conv. arb	106	1.02	0.51	2.11	1.71	-0.14	1.38	7.18	5.33	0.73	0.14	30.93	17.31	12.81	16.87
Fixed income	36	0.58	0.35	2.37	1.97	-2.27	2.30	17.10	15.98	0.82	0.21	19.59	16.53	11.13	21.64

Table 2
Description of the liquidity factors

This table presents summary statistics (Panel A) and pair-wise correlations (Panel B) for six determinants of time-varying liquidity over the sample period, January 1993 to August 2004.

Panel A: Summary statistics

	Mean	SD	Skewness	Kurtosis
Stock market return	0.76	4.28	-0.57	3.55
Stock market volatility	15.40	7.64	1.44	6.06
Stock market liquidity	-2.72	7.20	-0.97	6.07
Bond market return	0.03	1.16	-0.53	3.78
Bond market volatility	3.93	1.18	0.85	4.47
Bond market liquidity	0.05	0.02	0.16	1.13

Panel B: Pair-wise correlation among liquidity factors

	Stock mkt return	Stock mkt volatility	Stock mkt liquidity	Bond mkt return	Bond mkt volatility
Stock mkt volatility	-0.16				
Stock mkt liquidity	0.19	-0.17			
Bond mkt return	0.04	0.06	-0.07		
Bond mkt volatility	0.06	-0.01	0.02	-0.30	
Bond mkt liquidity	-0.15	0.48	-0.22	-0.01	0.03

Table 3
Determinants of hedge fund liquidity - general specification

This table shows the results of estimating the relation between liquidity in hedge fund returns and the nine factors listed for the eight strategies during the sample period January 1993 to August 2004. A positive parameter estimates indicates a positive relationship between the factor and the degree of autocorrelation in the fund return, and thus a negative relation between the factor and fund liquidity. The t-statistics are reported parentheses below the parameter estimates; parameters that are significantly different from zero at the 95% confidence level are in bold. All t-statistics and hypothesis tests are based on Newey-West (1987) robust standard errors. In the third-last and second-last rows we present the χ^2-statistics and p-values of joint tests that all time-varying liquidity factors have coefficients equal to zero, i.e. that the serial correlation (liquidity) of all hedge fund returns in a given style is constant. This statistic has the χ^2 distribution with nine degrees of freedom under the null hypothesis, and the 95% critical value is 16.92. The final row of the table presents the value of the log-likelihood at the optimum for each style.

Table 3
Determinants of hedge fund liquidity - general specification

See the previous page for a description of this table.

	Equity-based strategies					Non equity-based strategies		
Factor	Market neutral	Equity hedge	Equity non-hedge	Merger arbitrage	Distressed securities	Convertible arbitrage	Fixed income	Global macro
Stock market return	**−0.03**	**−0.05**	−0.09	−0.03	−0.02	−0.00	0.02	−0.02
	(−2.11)	**(−3.16)**	(−1.37)	(−1.22)	(−0.70)	(−0.11)	(0.66)	(−1.16)
Stock market volatility	0.00	−0.00	−0.01	0.01	0.03	**0.04**	**0.02**	−0.01
	(0.38)	(−0.18)	(−0.28)	(1.23)	(1.94)	**(4.30)**	**(2.21)**	(−0.47)
Stock market liquidity	0.01	0.01	0.02	−0.01	−0.01	0.00	−0.00	0.01
	(0.95)	(1.46)	(1.21)	(−1.39)	(−1.21)	(−0.35)	(−0.03)	(0.88)
Bond market return	**0.12**	**0.24**	0.13	0.06	0.07	0.03	−0.04	**0.18**
	(2.41)	**(2.90)**	(0.71)	(0.90)	(0.98)	(0.30)	(−0.83)	**(2.54)**
Bond market volatility	0.03	0.07	0.01	0.00	0.00	−0.04	0.06	0.05
	(0.58)	(1.05)	(0.10)	(0.03)	(0.03)	(−0.50)	(0.80)	(1.20)
Bond market liquidity	0.05	0.02	0.00	0.05	−0.03	**−7.77**	0.00	0.06
	(0.02)	(0.00)	(0.00)	(0.01)	(−0.01)	(−1.72)	(0.00)	(0.01)
Winter dummy	0.02	−0.08	0.16	0.14	0.18	−0.06	0.01	−0.02
	(0.19)	(−0.53)	(0.49)	(0.88)	(1.24)	(−0.27)	(0.06)	(−0.18)
Net fund flow	0.01	0.01	−9.59	**−0.10**	5.28	−0.09	**0.06**	**0.01**
	(0.29)	(0.27)	(−0.77)	**(−3.18)**	(0.80)	(−1.85)	**(2.16)**	**(1.98)**
Sign of "true return"	0.04	0.03	−0.00	−0.07	−0.05	−0.13	**−0.33**	0.06
	(0.60)	(0.37)	(−0.01)	(−0.81)	(−0.50)	(−1.25)	**(−3.05)**	(0.97)
χ^2-statistic	19.41	27.17	31.69	35.80	26.59	52.75	53.16	13.40
p-value	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.15
Log likelihood	-176.00	-104.38	-42.23	-143.18	-114.14	-120.23	-36.82	-148.88

Table 4
Determinants of hedge fund liquidity - reduced specification

This table shows the results of estimating the relation between liquidity in hedge fund returns and the nine factors listed for the eight strategies during the sample period January 1993 to August 2004. A positive parameter estimates indicates a positive relationship between the factor and the degree of autocorrelation in the fund return, and thus a negative relation between the factor and fund liquidity. The models presented are obtained from general-to-simple specification searches, where we started with the most general models, presented in Table 3, and then sequentially eliminated factors until all the factors remaining in the models were significant at 10% level. See the text for a more detailed description of our model selection algorithm. The t-statistics are reported parentheses below the parameter estimates; parameters that are significantly different from zero at the 95% confidence level are in bold. All t-statistics and hypothesis tests are based on Newey-West (1987) robust standard errors. In the third-last and second-last rows we present the χ^2-statistics and p-values of joint tests that all time-varying liquidity factors have coefficients equal to zero, i.e. that the serial correlation (liquidity) of all hedge fund returns in a given style is constant. This statistic has the χ^2 distribution with two or three degrees of freedom under the null hypothesis (depending on the number of factors remaining in the reduced model), and the 95% critical values are 5.99 and 7.81 respectively. The final row of the table presents the value of the log-likelihood at the optimum for each style.

Table 4
Determinants of hedge fund liquidity - reduced specification

See the previous page for a description of this table.

	Equity-based strategies					Non equity-based strategies		
Factors	Market neutral	Equity hedge	Equity non-hedge	Merger arbitrage	Distressed securities	Convertible arbitrage	Fixed income	Global macro
Stock market return	**−0.02** (−2.00)	**−0.03** (−2.55)	**−0.10** (−3.17)	–	–	–	–	–
Stock market volatility	–	–	–	–	**0.03** (2.65)	**0.04** (4.87)	**0.02** (3.18)	–
Stock market liquidity	–	–	**0.02** (2.57)	–	–	–	–	–
Bond market return	**0.11** (2.49)	**0.21** (3.36)	–	–	0.10 (1.92)	–	–	**0.15** (2.54)
Bond market volatility	–	–	–	–	–	–	–	–
Bond market liquidity	–	–	–	–	–	−7.90 (−1.82)	–	–
Winter dummy	–	–	–	**0.36** (2.99)	**0.26** (2.17)	–	–	–
Net fund flow	–	–	–	**−0.10** (−2.59)	–	**−0.09** (−2.35)	**0.05** (2.50)	0.01 (1.83)
Sign of "true return"	–	–	–	–	–	–	**−0.35** (−3.54)	–
χ^2-statistic	10.69	16.51	12.88	14.74	20.75	35.77	25.19	10.15
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
Log likelihood	-176.05	-104.47	-42.27	-143.59	-114.22	-120.39	-36.86	-148.96

Table 5
Predictors of hedge fund liquidity

This table shows the results of estimating the relation between liquidity in hedge fund returns and the nine factors listed for the eight strategies during the sample period January 1993 to August 2004. All factors in this specification are lagged by one month, meaning that this model may be used for predicting future hedge fund liquidity. A positive parameter estimates indicates a positive relationship between the factor and the degree of autocorrelation in the fund return, and thus a negative relation between the factor and fund liquidity. The t-statistics are reported parentheses below the parameter estimates; parameters that are significantly different from zero at the 95% confidence level are in bold. All t-statistics and hypothesis tests are based on Newey-West (1987) robust standard errors. In the third-last and second-last rows we present the χ^2-statistics and p-values of joint tests that all time-varying liquidity factors have coefficients equal to zero, i.e. that the serial correlation (liquidity) of all hedge fund returns in a given style is constant. This statistic has the χ^2 distribution with eight degrees of freedom under the null hypothesis, and the 95% critical value is 15.51. The final row of the table presents the value of the log-likelihood at the optimum for each style.

Table 5
Predictors of hedge fund liquidity

See the previous page for a description of this table.

	Equity-based strategies					Non equity-based strategies		
Lagged factors	Market neutral	Equity hedge	Equity non-hedge	Merger arbitrage	Distressed securities	Convertible arbitrage	Fixed income	Global macro
Stock market return	**−0.03**	**−0.05**	−0.04	**−0.03**	**−0.03**	**−0.04**	**−0.03**	**−0.05**
	(−2.67)	**(−3.84)**	(−1.28)	**(−2.06)**	**(−2.46)**	**(−2.82)**	**(−2.43)**	**(−3.85)**
Stock market volatility	0.00	−0.02	−0.03	**−0.01**	−0.00	−0.01	0.00	−0.01
	(0.46)	(−1.88)	(−1.53)	**(2.42)**	(−0.08)	(−0.67)	(0.06)	(−1.48)
Stock market liquidity	0.00	−0.00	−0.00	−0.01	**−0.02**	**−0.03**	**−0.04**	−0.01
	(0.07)	(−0.13)	(−0.26)	(−1.30)	**(−2.20)**	**(−3.65)**	**(−3.86)**	(−1.32)
Bond market return	−0.08	−0.06	−0.12	−0.03	−0.07	**−0.18**	0.11	−0.12
	(−1.72)	(−0.99)	(−1.09)	(−0.58)	(−0.89)	**(−2.93)**	(1.70)	(−1.83)
Bond market volatility	0.03	−0.09	−0.04	−0.03	−0.04	**−0.14**	−0.07	0.01
	(0.62)	(−1.23)	(−0.31)	(−0.56)	(−0.44)	**(−2.10)**	(−1.56)	(0.05)
Bond market liquidity	−0.03	−0.02	−0.01	**−9.79**	0.01	0.03	0.04	−2.12
	(−0.01)	(−0.00)	(−0.00)	**(−1.99)**	(0.00)	(0.01)	(0.01)	(−0.19)
Winter dummy	0.05	−0.15	0.23	**0.33**	0.29	0.09	0.20	−0.01
	(0.49)	(−1.05)	(1.06)	**(3.11)**	(1.70)	(0.60)	(1.90)	(−0.02)
Net fund flow	−0.00	**0.00**	0.05	0.00	0.00	**−0.00**	0.08	**0.00**
	(−0.20)	**(4.01)**	(0.71)	(0.84)	(0.02)	**(−3.90)**	(1.57)	**(1.96)**
χ^2-statistic	19.47	59.99	12.88	38.76	25.71	51.60	57.09	38.23
p-value	0.01	0.00	0.12	0.00	0.00	0.00	0.00	0.00
Log likelihood	-176.89	-103.78	-42.01	-142.67	-113.85	-119.21	-36.41	-149.45

Table 6
Determinants of hedge fund liquidity - robustness checks

This table shows the results of estimating the relation between liquidity in hedge fund returns and the nine factors listed for five categories of hedge funds, during the sample period January 1993 to August 2004. "Live" funds are those that were still reporting to the database as at the end of our sample, while "dead" funds are those that had ceased reporting. "Open" funds are those that were open to new money as at the end of our sample period, while "closed" funds were not accepting new money. "Non-audited" funds that had not yet been audited as at the end of our sample period. In all cases we only consider with at least 48 observations. A positive parameter estimates indicates a positive relationship between the factor and the degree of autocorrelation in the fund return, and thus a negative relation between the factor and fund liquidity. The t-statistics are reported parentheses below the parameter estimates; parameters that are significantly different from zero at the 95% confidence level are in bold. All t-statistics and hypothesis tests are based on Newey-West (1987) robust standard errors. In the second-last and last rows we present the χ^2-statistics and p-values of joint tests that all time-varying liquidity factors have coefficients equal to zero, i.e. that the serial correlation (liquidity) of all hedge fund returns in a given style is constant. This statistic has the χ^2 distribution with nine degrees of freedom under the null hypothesis, and the 95% critical value is 16.92.

Table 6
Determinants of hedge fund liquidity - robustness checks

See the previous page for a description of this table.

	Live funds	Dead funds	Open funds	Closed funds	Non-audited funds
Number of funds	361	248	308	48	20
Factors					
Stock market return	−0.03	−0.01	−0.02	−0.03	−0.03
	(−1.65)	(−0.96)	(−1.55)	(−1.63)	(−1.63)
Stock market volatility	0.01	0.01	0.01	0.03	0.01
	(1.37)	(1.68)	(1.18)	(1.88)	(0.42)
Stock market liquidity	0.01	−0.01	0.00	0.02	0.02
	(0.60)	(−0.70)	(0.48)	(1.87)	(1.60)
Bond market return	**0.10**	**0.15**	**0.10**	**0.14**	0.06
	(2.04)	**(2.25)**	**(1.96)**	**(2.42)**	(0.60)
Bond market volatility	0.02	0.00	0.01	−0.01	−0.09
	(0.31)	(0.00)	(0.28)	(−0.10)	(−1.20)
Bond market liquidity	0.32	0.09	0.03	0.01	0.02
	(0.09)	(0.02)	(0.01)	(0.00)	(0.00)
Winter dummy	0.06	0.00	0.07	0.00	0.12
	(0.56)	(0.02)	(0.61)	(0.02)	(0.74)
Net fund flow	0.00	0.00	0.00	0.00	0.00
	(0.08)	(0.92)	(0.63)	(1.71)	(1.36)
Sign of "true return"	−0.02	−0.05	−0.03	0.01	−0.01
	(−0.34)	(−0.66)	(−0.42)	(0.13)	(−0.10)
χ^2-statistic	18.21	25.99	18.71	31.99	16.34
p-value	0.03	0.00	0.03	0.00	0.06

Table 7: Finite-sample size of t-tests on individual factor coefficients

This table presents the proportion of t-statistics on the coefficients on individual factors that were outside the 95% confidence interval (± 1.96) across the simulation replications. As $T \to \infty$ these proportions asymptote to 0.05, the nominal size of this test. The simulation design is described in the body of the text. These results are based on 300 replications.

		No missing obs, no 'true return' factor			With missing obs and 'true return' factor		
Number of funds:		**K = 1**	**K = 10**	**K = 50**	**K = 1**	**K = 10**	**K = 50**
Sample size	*Number of factors*						
$T = 75$	*1*	0.09	0.09	0.06	0.10	0.07	0.03
$T = 150$	*1*	0.05	0.06	0.06	0.07	0.04	0.04
$T = 500$	*1*	0.02	0.09	0.04	0.03	0.05	0.04
$T = 75$	*4*	0.13	0.07	0.05	0.16	0.05	0.02
$T = 150$	*4*	0.08	0.06	0.05	0.09	0.05	0.03
$T = 500$	*4*	0.04	0.06	0.06	0.03	0.05	0.05
$T = 75$	*8*	0.23	0.08	0.06	0.11	0.06	0.04
$T = 150$	*8*	0.08	0.07	0.06	0.10	0.05	0.04
$T = 500$	*8*	0.03	0.06	0.06	0.03	0.05	0.05

Table 8: Finite-sample size of χ^2 joint tests on factor coefficients

This table presents the proportion of χ^2-statistics that were larger than the 95% critical value of the appropriate χ^2 distribution across the simulation replications. (The degrees of freedom is equal to the number of factors.) As $T \to \infty$ these proportions asymptote to 0.05, the nominal size of this test. The simulation design is described in the body of the text. These results are based on 300 replications.

		No missing obs, no 'true return' factor			With missing obs, and 'true return' factor		
Number of funds:		**K = 1**	**K = 10**	**K = 50**	**K = 1**	**K = 10**	**K = 50**
Sample size	*Number of factors*						
$T = 75$	*1*	0.09	0.09	0.06	0.17	0.05	0.03
$T = 150$	*1*	0.05	0.06	0.06	0.06	0.03	0.01
$T = 500$	*1*	0.02	0.09	0.04	0.02	0.02	0.02
$T = 75$	*4*	0.33	0.16	0.08	0.73	0.17	0.08
$T = 150$	*4*	0.19	0.09	0.07	0.41	0.10	0.03
$T = 500$	*4*	0.04	0.07	0.09	0.08	0.05	0.04
$T = 75$	*8*	0.78	0.33	0.21	0.98	0.45	0.18
$T = 150$	*8*	0.45	0.19	0.12	0.79	0.16	0.06
$T = 500$	*8*	0.05	0.07	0.06	0.17	0.06	0.06

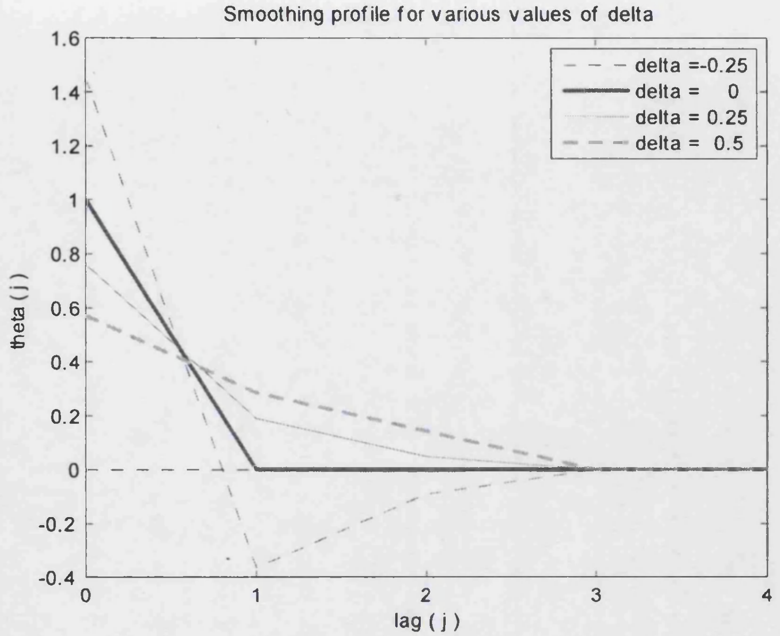

Figure 1: *This figure shows the relationship between the MA coefficients, θ_j, and the smoothing parameter, δ, for various values of δ.*

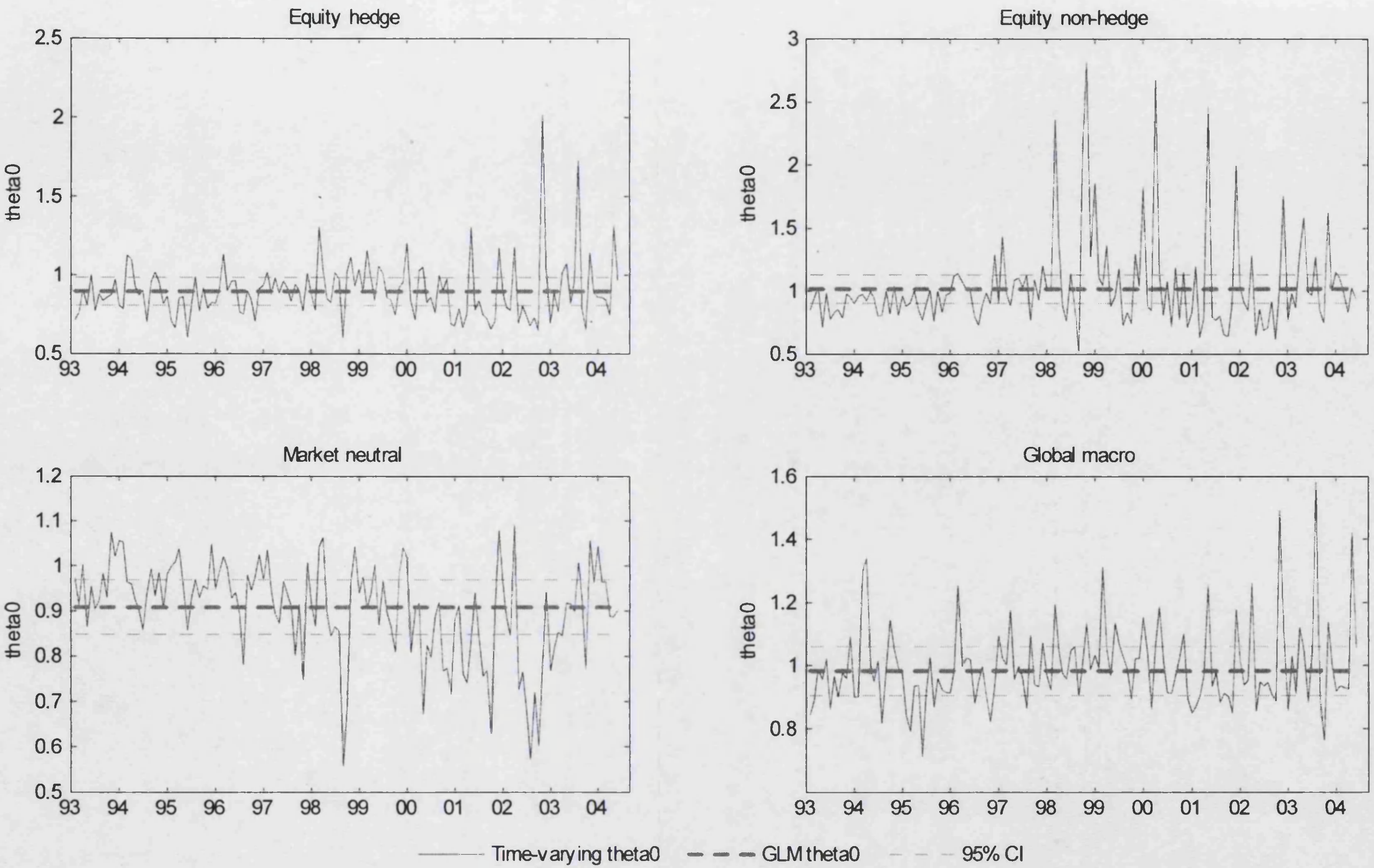

Figure 2: *This figure plots the estimated fraction of the true return at time t revealed in the observed return at time t, denoted $\hat{\theta}_0$, based on the model for time-varying liquidity and from GLM's constant $\hat{\theta}_0$ model. The styles presented are Equity hedge, Equity non-hedge, Market neural, and Global macro.*

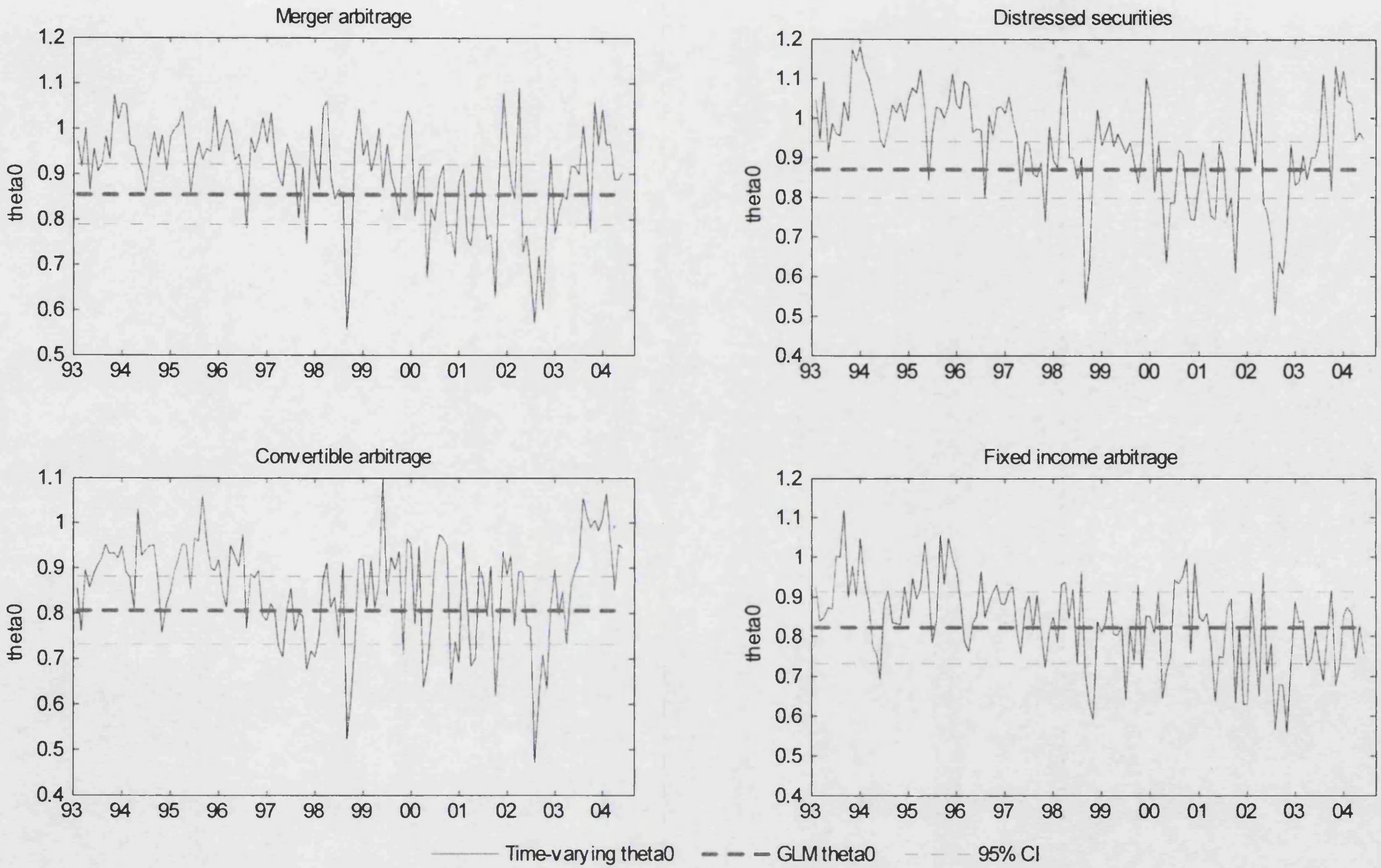

Figure 3: *This figure plots the estimated fraction of the true return at time t revealed in the observed return at time t, denoted $\hat{\theta}_0$, based on the model for time-varying liquidity and from GLM's constant $\hat{\theta}_0$ model. The styles presented are Merger arbitrage, Distressed securities, Convertible arbitrage, and Fixed income arbitrage.*

Chapter 2

Market Dispersion and the Profitability of Hedge Funds

Market Dispersion and the Profitability of Hedge Funds*

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August 2007

Abstract

We examine the impact of market dispersion on the performance of hedge funds using hedge fund indices and a panel of monthly returns on over 600 individual hedge funds. Market dispersion is measured by cross-sectional volatility, that is, the standard deviation across all asset returns in one time period. We exploit the information held in the cross-sectional dispersion of equity returns and find that market dispersion and the performance of hedge funds are positively related across all equity-oriented hedge funds. Containing information very different from other factors, cross-sectional volatility is an important determinant of hedge fund returns. We also find the level of hedge fund return dispersion is positively related to the level of market dispersion.

Keywords: Market Dispersion; Hedge Fund Performance

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1 Introduction

Understanding risk exposure of hedge funds has become an increasingly important area of research since the hedge fund industry has grown to be one of the most important segments of the financial services industry. The assets under management by hedge funds is estimated to have surpassed $1.4 trillion at the end of 2006 and, when combined with leverage-intensive strategies, this is large enough to move markets around the world. Risk modeling is an essential tool for hedge fund managers because they often take sizable active bets and use extensive leverage. These models can help them to understand the risks taken to achieve previous returns and to forecast future risk so that managers are able to accurately balance return against risk. Hedge fund investors need a better understanding of hedge funds' risk exposure when they make investment management decisions involving hedge funds. Regulators are also concerned about the potential damage hedge funds can cause to stability of financial markets.

A number of recent papers have studied the risk exposures and returns of hedge funds. Using a variety of hedge fund databases, Ackermann, McEnally, and Ravenscraft (1999), Agarwal and Naik (2000), Edwards and Caglayan (2001), Fung and Hsieh (1997, 2002) and Liang (1999, 2000, 2001) offer comprehensive studies of historical hedge fund performance. Another strand of literature focuses more on the characteristic of risk and return in specific hedge fund strategies. For example, Mitchell and Pulvino (2001) study the "risk-arbitrage" strategy; Fung and Hsieh (2001) focus on the "trend following" strategy and Agarwal and Naik (2004) study a number of equity-oriented strategies. However, none of these studies focuses on hedge funds' exposure to market volatility risk.

Some related studies examine the issue whether hedge funds amplify market volatilities and impair stability of financial markets (see, Eichengreen et al., 1998, Fung and Hsieh, 2000), but they do not examine whether hedge funds benefit from volatile financial markets, i.e, whether hedge funds exhibit systematic exposure to volatility risk. Bondarenko (2004) estimates the value of the variance contract from prices of traded options and finds that the variance return is a key determinant in explaining performance of hedge funds. Most hedge funds exhibit negative exposure to the variance return, implying that they actually be short in volatility owing to the nature of their strategies. Hence, the performance of hedge funds tends to be worse when markets are volatile. This seems to contradict the conventional wisdom that hedge funds thrive in volatile financial markets. As alternative investment tools the primary benefit to hedge fund investing is the low correlation between returns of hedge funds and of traditional asset classes[1].

In this paper, we provide a comprehensive empirical investigation of the impact of market dispersion on the profitability of hedge funds. We use cross-sectional volatility to measure the magnitude of market dispersion. In contrast to time series market volatility, which is a measure of the variation of market returns over a period of time, cross-sectional market volatility is the standard deviation across all asset returns at one time period. Time series market volatility is the volatility experienced by holders of aggregate index funds and it is the measure for market risk which is nondiversifiable. There is an extensive literature on time series market volatility (please see Bollerslev, Chou and Kroner (1992), Ghysels, Harvey and Renault (1996) for the summary of some literature). On the contrary, cross-sectional volatility captures the cross-sectional dispersion of asset returns within a market at one time period. For equity markets, the dynamics of cross-sectional volatility are of economic importance to hedge funds for several reasons. First, it is a good measure of opportunities available to hedge funds for generating active returns because it helps describe how the environment for strategies driven by stock, or sector selection is changing: if stock returns were all the same, their cross-sectional volatility would be zero and there would be no opportunity to produce active returns. As the cross-sectional volatility increases, so does the opportunity to out-perform/under-perform a benchmark. Therefore, if cross-sectional volatility is interpreted as a means to measure the opportunities for fund managers to add value, high cross-sectional volatility in markets probably means more opportunities for active fund managers to outperform the market.

[1] Patton (2007) proposes generalizing the concept of "market neutrality" to consider the "completeness" of the fund's neutrality to market risks. "Complete neutrality" corresponds to independence of the fund and the market returns. He finds that about one-quarter of funds in the "market neutral" category are not in fact market neutral.

Second, cross-sectional volatility can also be regarded as a proxy for idiosyncratic risk because it is a measure of heterogeneity across the securities in the market, and the heterogeneity of security returns is driven by idiosyncratic shocks to these securities. Campbell, Lettau, Malkiel and Xu (2001) decompose the total volatility of a stock into three components, market volatility, industry volatility, and firm specific (idiosyncratic) volatility. Cross-sectional volatility can be viewed as the weighted average of idiosyncratic stock volatility. If cross-sectional volatility is related to the performance of hedge funds, it means that idiosyncratic risk matters for hedge funds. The possible rationale is that as highly sophisticated investors, hedge funds seek to exploit pricing inefficiencies between related individual securities, hence risks that they face are related to idiosyncratic stock volatility, not aggregate market volatility. When idiosyncratic volatility is high larger pricing errors become possible (Ingersoll 1987, Chapter 7, Shleifer and Vishny 1997). Whether idiosyncratic risk is a determinant of hedge fund performance is also directly related to the debate in empirical finance of whether idiosyncratic risk is priced in the equities market.

As a first step in our investigation, we estimate the cross-sectional volatility of equity returns across time using CRSP data from January of 1994 to December of 2004. Each month, we use all the stocks which have a valid return for that month. Consistent with previous studies, the cross-sectional volatility of equity returns varies significantly over the sample period and is serially correlated. We then test the hypothesis that cross-sectional volatility affects the profitability of hedge funds using hedge fund indices from Hedge Fund Research (HFR). HFR databases classifies hedge funds into several categories according to their investment strategies. To examine incremental explanatory value of cross-sectional volatility to hedge fund returns, we conduct a careful analysis on risk adjustments for hedge fund returns to obtain hedge fund abnormal performance. Many studies have shown that due to the dynamic trading strategies and derivatives used by hedge funds, traditional linear asset pricing models could give misleading results on hedge fund performance. We use the seven-factor model of Fung and Hsieh (2004). These factors have been shown to have considerable explanatory power for hedge fund returns, and are studied in the third chapter of this thesis.

The results at the hedge fund index level strongly suggest that there is a genuine relation between cross-sectional volatility and the performance of hedge funds. To examine this issue in a more structured way, we provide parametric joint (cross-fund) tests using the individual hedge fund return data from the Center for International Securities and Derivatives Markets (CISDM) hedge fund database. Since it is well documented that hedge fund returns exhibit significant serial correlation, we estimate a pooled regression model with panel corrected standard errors (PCSE). Our PCSE specification allows errors to be contemporaneously correlated, heteroskedastic across funds and autocorrelated within each fund's time series. We report the results of several such joint tests. We find a highly statistically significant positive relation between cross-sectional dispersion of equity returns and hedge fund returns.

We further investigate how the dispersion of hedge fund returns corresponds to the dispersion of market. Silva, Sapra and Thorley (2001) find that the wide dispersion in security returns has led to wide dispersion in mutual fund returns. Consistent with findings in mutual funds, we find that the level of hedge fund return dispersion is also positively related to the level of market dispersion. This has important implications for hedge fund performance evaluation.

Overall, our results suggest that the cross-sectional volatility can explain part of hedge fund returns. Cross-sectional volatility contains information very different from other factors, and it is an important determinant of fund returns. We also extend the literature on idiosyncratic risk by studying the exposure of hedge funds to the average idiosyncratic risk.

The remainder of the paper is organized as follows. Section 2 discusses related literature. Section 3 presents the measures of cross-sectional volatility and links market dispersion to the performance of hedge funds. Section 4 discusses implications for portfolio management. Section 5 concludes.

2 Related literature

While time series volatility has been extensively studied, little study has been done on cross-sectional volatility. Hwang (2001) compares the properties of cross-sectional volatility with those of time-series market volatility such as squared market returns in the UK and US markets. His empirical results show that cross-sectional market volatility is highly correlated with time-series market volatility and contains more information about the market evolution than squared market returns.

Since cross-sectional volatility can be regarded as a proxy for idiosyncratic risk, another closely related literature is the relation between idiosyncratic risk and stock returns. The asset pricing literature does not have consensus in the cross-sectional role of idiosyncratic risk in stocks. According to the traditional CAPM theory, only market risk should be priced in equilibrium and investors will not be rewarded for taking idiosyncratic risk because it can be diversified away. However, Levy (1978), Merton (1987), and Malkiel and Xu (2002) extend the CAPM. In their models, investors may hold undiversified portfolios for some exogenous reasons and idiosyncratic risk is priced in equilibrium. For example, in reality institutional investors may take certain idiosyncratic risk in order to obtain extraordinary returns. Hence, investors will care about total risk not just the market risk.

Empirical work provides mixed evidence for the role of idiosyncratic risk in asset pricing. On the one hand, Lintner (1965), Tinic and West (1986), Lehmann (1990) and Malkiel and Xu (2002) find there is a positive relation between idiosyncratic volatility and stock returns. On the other hand, Longstaff (1989) finds that a cross-sectional regression coefficient on total variance for size-sorted portfolios has an insignificant negative sign. Ang, Hodrick, Xing and Zhang (2006) find stocks with high idiosyncratic volatility have very low average returns. Hirt and Pandher (2005) find idiosyncratic volatility is negatively priced in risk-adjusted stock returns but its effect is not significant in unadjusted returns.

At the aggregate level of idiosyncratic risk which can be measured by cross-sectional volatility, Goyal and Santa-clara (2003) show that the effects of idiosyncratic risk is diversified away in the equal-weighted portfolio variance measure, but it makes up almost 85% of the equal-weighted average stock variance. Their average stock variance can be interpreted as a measure of cross-sectional dispersion of stock returns. Goyal and Santa-clara find a significant positive relation between average stock variance (largely idiosyncratic) and the return on the market. However, Bali, Cakici,Yan and Zhang (2005) show that this result is driven by small stocks traded on the Nasdaq and it does not hold for the extended sample from1963:08 to 2001:12 and for the NYSE/AMEX and NYSE stocks.

Market dispersion captured by cross-sectional volatility is also linked to the dispersion of fund returns. Silva, Sapra and Thorley (2001) find that the wide dispersion in security returns has led to wide dispersion in mutual fund returns. This wide dispersion in mutual fund returns has little to do with changes in the informational efficiency of the market or the range of managerial talent. Through correcting fund alphas with a period- and asset-class-specific measure of security return dispersion, they extend performance benchmarking to incorporate the information embedded in return dispersion. Ankrim and Ding (2002) demonstrate that changes in the level of cross-sectional volatility have a significant association with the distribution of active manager returns.

Cross-sectional volatility has also been studied under the different context. Christie and Huang (1995) use cross-sectional volatility to capture herd behavior in stock markets. Bessembinder, Chan, and Seguin (1996) use cross-sectional dispersion of stock returns as a proxy for company-specific information flows. Solnik and Roulet (2000) argue that dispersion is a better measure of the benefits of diversification than correlation. They show the relationship between correlation, dispersion and the standard deviation of the market portfolio.

The literature on hedge fund performances has increased rapidly. Fung and Hsieh (2001) study the "trend-following" strategy and find that equity-oriented hedge funds have non-linear, option-like payoffs with respect to the market return. Mitchell and Pulvino (2001) analyze the "risk-arbitrage" strategy. Agarwal and Naik (2004) study a number of equity-oriented strategies. In particular, they include call and put options on the S&P 500 composite index as risk factors.

3 Empirical analysis

In this section, we introduce our dispersion measures and investigate their relation to the performance of hedge funds. Fundamental to hedge fund investment process is a proper assessment of the risk and return potential of an investment strategy. The dynamics of cross-sectional return dispersion over time is an important factor in determining attractiveness of investment strategies.

3.1 Data

Cross-sectional dispersion can be described by a number of different measures, such as range, inter-quartile range and mean absolute deviation. Cross-sectional volatility is an attractive proxy for dispersion, since it takes into account the entire collection of securities returns within a market at one time period. We define equal-weighted cross-sectional volatility as:

$$CSV_t = \sqrt{\frac{1}{N_t}\sum_{i=1}^{N_t}(r_{it} - r_{ew})^2} \tag{1}$$

where r_{it} is the observed stock return on firm i at time t and r_{ew} is the cross-sectional average of the N returns in the aggregate market portfolio at time t. This dispersion measure quantifies the average proximity of individual returns to the realized market average.

We compute cross-sectional volatility measure using CRSP data from January of 1994 to December of 2004. Each month, we use all the stocks which have a valid return for that month.

As in French, Schwert and Stambaugh (1987), the estimate of the monthly stock market volatility is:

$$MV_t = \sqrt{\sum_{i=1}^{N_t} r_{it}^2 + 2\sum_{i=1}^{N_t-1} r_{it} r_{i+1,t}} \tag{2}$$

where r_{it} is the return to the S&P portfolio on day i in month t and there are N_t trading days in month t.

Panel A of Table 1 gives descriptive statistics on the volatility measures. Both volatility measures are highly persistent and show a substantial time-variation.

Panel B of Table 1 reports the correlations among two volatility measures and other risk factors of hedge fund returns. As we can see from the table, cross-sectional volatility and time series market volatility is positively correlated. Small cap minus large cap return is also positively correlated with cross-sectional volatility.

Figure 1 shows the evolution of cross-sectional dispersion measure of US assets, together with time series market volatility. There is a steady increase in the cross-sectional volatility of monthly returns through the 1990s. During the Internet Bubble period 1999 to 2001, cross-sectional volatility increases sharply and reaches its peak. Since then, cross-sectional volatility has declined dramatically, falling below pre-bubble levels in 2004.

3.2 Performance of hedge funds

We analyse performance of hedge funds both at index level and individual fund level. We first focus on indices from HFR database. The appendix provides a brief description of the sector indices. We consider all main fund categories for which data are available from the database inception in 1990. Those include 7 equity categories: Convertible Arbitrage, Distressed Securities, Equity Hedge, Equity Market Neutral, Equity Non-Hedge, Event-Driven, Market Timing; 1 aggregate categories: Fund of Funds Composite. Table 2 reports summary statistics of monthly returns of HFR index.

For individual hedge fund returns, we use CISDM hedge fund database, maintained by the University of Massachusetts in cooperation with Managed Account Reports LLC, with data through August 2004. The CISDM database consists of two sets of files, one for live funds and one for dead funds. Each set consists of a performance file, containing monthly observations of returns, total net assets, and net asset values, and a fund information file, containing fund name, strategy type, management fees, and other supplementary details. We discard funds with less than 48 months of returns.

We single out Equity Hedge, Equity Nonhedge, Market Neutral, Merger Arbitrage, Distressed Securities[2] and Convertible Arbitrage for further scrutiny. Equity hedge funds have grown considerably over time (now representing the single largest strategy according to

[2]In CISDM database, event driven style includes merger arbitrage and distressed securities.

HFR) and have the highest alpha in Agarwal and Naik (2004). Another large sector of equity-oriented hedge funds is market neutral funds. Market Neutral strategies aim at zero exposure to specific equity market factors.

Table 3 presents summary statistics. For each strategy, the table lists the number of funds and means and standard deviations of basic summary statistics.

3.3 Cross-sectional market volatility and performance of hedge funds

We now explore the linkage between cross-sectional volatility and performance of hedge funds. To ensure robust findings, first we need to obtain risk-adjusted hedge fund returns. However, there is no well-established method for hedge fund risk adjustments in the existing literature due to their use of derivatives and dynamic trading strategies. Here we use as performance benchmarks the seven-factor model developed by Fung and Hsieh (2004). The Fung and Hsieh (2004) factors are S&P 500 return minus risk free rate, Wilshire small cap minus large cap return, change in the constant maturity yield of the 10-year Treasury, change in the spread of Moody's Baa minus the 10-year Treasury, bond PTFS, currency PTFS, and commodities PTFS, where PTFS denotes primitive trend following strategy. Fung and Hsieh (2004) show that their factor model strongly explains variation in individual hedge fund returns.

In order to obtain risk-adjusted performance of hedge funds, we regress the net-of-fee monthly excess return (in excess of the risk free rate) of a hedge fund on the seven-factor model.

$$R_{i,t} = \alpha_i + \beta_i F_t + \epsilon_{i,t}, \tag{3}$$

where $R_{i,t}$ is the net-of-fee monthly excess return of fund i in month t, β_i represents the risk exposure of fund i at month t to the various factors, and F_t is the value of the various factors at month t. The risk-adjusted return of fund i at month t is calculated as:

$$\hat{a}_{i,t} = R_{i,t} - \hat{\beta}_i^{\top} F_t = \hat{\alpha}_i + \hat{\epsilon}_{i,t}, \tag{4}$$

where $\hat{\beta}_i$ is the estimated risk exposure for fund i. We compute the risk-adjusted returns $\hat{a}_{i,t}$ as the sum of the intercept $\hat{\alpha}_i$ and the residual $\hat{\epsilon}_{i,t}$ of Eq.(3).

To explore the relation between cross-sectional volatility and hedge fund performance, the empirical analysis is mainly based on the following regression:

$$\hat{a}_t = \beta_0 + \beta_1 CSV_t + \beta_2 MV_t + \epsilon_t \tag{5}$$

where $\hat{a}_t$ is risk-adjusted hedge fund return index at month t, CSV_t is cross-sectional volatility at month t, MV_t is time series volatility at month t from equation 2.

Table 4 reports the regression results. The exposure to cross-sectional volatility is positive and significant for convertible arbitrage, equity hedge and equity non-hedge funds. All three strategies involves long and short in equity markets. The sensitivity to time series volatility is negative and insignificant for all funds except equity hedge funds. It is probably not surprising that event driven, market timing and fund of funds do not have significant exposure to cross-sectional volatility. Event driven funds involve investing in opportunities created by significant transactional events and the performance of market timing funds depends on fund managers' ability to identify market uptrend and downtrend. Funds of funds invest in a group of other hedge funds, hence funds of funds actually diversify away idiosyncratic risks in individual hedge funds.

While the hedge fund index results reported above strongly suggest that hedge fund performances are correlated with market dispersion, we can use the entire hedge fund database to make more definitive statements about the statistical significance of the market dispersion effect.

The joint tests can be considered simple generalizations of the regressions described above. Using all of the data from individual funds, we estimate a pooled regression model of the form:

$$\hat{a}_{i,t} = c_i + b_1 CSV_t + b_2 MV_t + u_{i,t} \quad (6)$$

with panel corrected standard errors (PCSE). Where $\hat{a}_{i,t}$ is the estimated risk-adjusted return of fund i at month t. The parameters b_1 and b_2 are constrained to be the same across funds. Our PCSE specification allows $u_{i,t}$ to be contemporaneously correlated and heteroskedastic across funds, and autocorrelated within each fund's time series.

We report the results of joint (across funds) tests of significance in Table 5. Consistent with previous results, equity hedge and convertible arbitrage exhibit significant positive exposure to cross-sectional volatility. The difference is that the performance of market neutral funds now is significantly positively correlated to cross-sectional volatility. This might be due to the use of different databases or the extra gain from cross-sectional data by pooling these funds within the same style.

In summary, consistent with the results of Bondarenko (2004), we find market volatility is negatively related to hedge fund performance across all strategies. For equity hedge and merger arbitrage, this negative relation is also significant. These results suggest that higher volatility was associated with lower hedge fund returns and many hedge funds could be short in volatility due to the nature of their strategies. Hedge funds may have a negative exposure to the market volatility risk in several ways. First, many equity-oriented hedge funds take short position on volatility through variance swaps, a forward contract on future realized price variance, or through dispersion trades whose payoff is equal to the difference between the variance of an index and its component stocks. The dispersion strategy typically consists of short selling options on a stock market index while simultaneously buying options on the component stocks, i.e. short market volatility and long dispersion. Second, equity-oriented hedge funds usually take bets on events such as mergers, spin-offs, takeovers, corporate restructuring, and reorganization. These strategies involve the risk of deal failure and are short the volatility risk because deals are more likely to fail in volatile markets than in normal markets.

The regression results also strongly suggest that the performance of equity style hedge funds is positively correlated with market dispersion. Cross-sectional volatility contains information very different from other factors which determine hedge fund returns. It is quite important for fund managers to take into account cross-sectional volatility. Suppose a talented fund manager has the real ability for picking up stocks and take long position on stock A and short position on stock B. When cross-sectional volatility increases, the difference between the movements of prices of A and B increases as A goes up and B goes down, so does the profit of the manager's trading position. Another example is dispersion trading. Typically long dispersion means short selling options on a stock market index and buying options on the component stocks. When dispersion increases, individual stock volatility increases, and dispersion trading makes a profit.

3.4 Market dispersion and dispersion of hedge fund returns

Dierick and Garbaravicius (2005) argue that the decreasing dispersion of hedge fund returns could be a broad indication that hedge fund positioning is becoming increasingly similar. Patterns in pairwise correlation coefficients of individual hedge fund return performance within strategies also indicate that hedge fund positioning has resulted in a crowding of trades in some markets, possibly leaving them vulnerable to adverse market dynamics. These concerns are the greatest for convertible arbitrage and credit strategies, as these strategies generally have the highest leverage and therefore significant gross positions. This sub-section examines these issues by analysing the relationship between market dispersion and the dispersion of hedge fund returns.

To investigate the impact of market dispersion on the dispersion in hedge fund returns, we regress cross-sectional dispersion in hedge fund returns on cross-sectional volatility of stock returns.

$$CSVR_t = \gamma_0 + \gamma_1 CSV_t + \gamma_2 STR_t + \gamma_3 SMB_t + \gamma_4 CSVR_{t-1} + \eta_t \qquad (7)$$

where $CSVR_t$ is cross-sectional dispersion of hedge fund returns at month t, CSV_t is cross-sectional volatility of stock returns at month t, STR_t is stock market return at month t, SMB_t is small minus big portfolio return at month t. To control for serial correlation of $CSVR_t$, we also include one lag of the dependent variable in the regression. Table 6 reports the regression results.

The results indicate that the dispersion in hedge fund returns is positively related to market dispersion. Therefore, the decreasing dispersion of hedge fund returns does not necessarily mean that hedge fund positioning is becoming increasingly similar. An increase in dispersion of hedge fund returns does not necessarily result from managers's more diverse skill levels, but possibly from the magnified riskiness of bets taken by fund managers. Hedge fund managers construct portfolios to beat a particular benchmark by holding securities in proportions that are different from those in the benchmark. The larger the differences between the portfolio weights and the benchmark weights, the greater the active risks the manager is taking. The level of the active risk is influenced by variability of securities returns captured by cross-sectional volatility.

4 Implication for portfolio management

The dynamics of cross-sectional volatility have important implications for portfolio management. The decrease in cross-sectional volatility will makes it more difficult for hedge fund managers to generate a spread between their short and long positions, which means fewer opportunities to outperform benchmarks. During periods of low cross-sectional volatility, it is important for hedge fund managers to understand how and to what extent opportunities can be preserved. In order to maintain a constant level of active risk – and therefore preserve the potential for active return – a hedge fund manager must increase active exposures as cross-sectional volatility declines.

The cross-sectional dispersion of asset returns also has important implications for funds' performance evaluation. A realized alpha of 10% generated during low cross-sectional volatility period probably means more to the investors than it does during the Internet Bubble period. Silva, Sapra and Thorley (2001) argue that an assessment of the performance of money managers should take into account the dispersion of stock returns during the period.

They find that the increase in the dispersion of portfolios of money managers in 1999 is the result of an increase in the dispersion of the underlying stocks rather than an increase in the diversity of manager talents or a decrease in market efficiency.

5 Concluding remarks

In this paper, we study the impact of market dispersion on the performance of hedge funds. First, we estimate the cross-sectional dispersions of equity returns. Using U.S. stock market data over the 1994-2004 period, we analyze the time-series properties of the market dispersions. We find that cross-sectional volatilities are time-varying and persistent. The fluctuations in the cross-sectional dispersion are positively related to the performance of equity-based hedge funds. We show that market dispersion, proxied by cross-sectional volatility, may explain part of the hedge fund returns not accounted for by the standard factors. Cross-sectional volatility should be considered as a new risk factor for equity-based hedge funds.

Our findings have important implications for hedge fund portfolio management and performance evaluation. During periods of high cross-sectional volatility, many hedge funds may deliver statistically positive risk-adjusted returns (alpha) if the cross-sectional volatility exposure is not taken into account. However, after correcting for cross-sectional volatility exposure, the performance of some hedge funds may become less impressive, with positive alphas becoming negative or statistically insignificant.

Our paper raises some interesting issues. Cross-sectional volatility can be regarded as a proxy of aggregate idiosyncratic risk. An interesting direction for future research would be to examine the determinants of idiosyncratic risk and how it changes over time. If we could understand the changes in cross-sectional volatility, we will probably have a better understanding of hedge funds' risk exposure.

6 Appendix:

HFR Monthly Performance indices

Hedge Fund Research (HFR) Monthly indices are equally weighted performance indices. The indices are broken down into several categories by strategy, including the HFR Composite Index, which accounts for over 1,400 funds listed on the internal HFR Database.

The following classification of hedge fund categories is reproduced from the HFR website.

Equity Hedge funds invest in a core holding of long equities hedged at all times with short sales of stocks and/or stock index options. Some managers maintain a substantial portion of assets within a hedged structure and commonly employ leverage. Where short sales are used, hedged assets may be comprised of an equal dollar value of long and short stock positions. Other variations use short sales unrelated to long holdings and/or puts on the S&P 500 index and put spreads. Conservative funds mitigate market risk by maintaining market exposure from zero to 100 percent. Aggressive funds may magnify market risk by exceeding 100 percent exposure and, in some instances, maintain a short exposure. In addition to equities, some funds may have limited assets invested in other types of securities.

Equity Market Neutral funds seek to profit by exploiting pricing inefficiencies between related equity securities, neutralizing exposure to market risk by combining long and short positions. One example of this strategy is to build portfolios made up of long positions in the strongest companies in several industries and taking corresponding short positions in those showing signs of weakness.

Equity Non-Hedge funds are predominately long equities although they have the ability to hedge with short sales of stocks and/or stock index options. These funds are commonly known as "stock-pickers." Some funds employ leverage to enhance returns. When market conditions warrant, managers may implement a hedge in the portfolio. Funds may also opportunistically short individual stocks. The important distinction between equity non-hedge funds and equity hedge funds is equity non-hedge funds do not always have a hedge in place. In addition to equities, some funds may have limited assets invested in other types of securities.

Event-Driven is also known as "corporate life cycle" investing. This involves investing in opportunities created by significant transactional events, such as spin-offs, mergers and acquisitions, bankruptcy reorganizations, recapitalizations and share buybacks. The portfolio of some Event-Driven managers may shift in majority weighting between Risk Arbitrage and Distressed Securities, while others may take a broader scope. Instruments include long and short common and preferred stocks, as well as debt securities and options. Leverage may be used by some managers. Fund managers may hedge against market risk by purchasing S&P put options or put option spreads.

Market Timing funds allocate assets among investments by switching into investments that appear to be beginning an uptrend, and switching out of investments that appear to be starting a downtrend. This primarily consists of switching between mutual funds and money markets. Typically, technical trend-following indicators are used to determine the direction of a fund and identify buy and sell signals. In an up move "buy signal," money is transferred from a money market fund into a mutual fund in an attempt to capture a capital gain. In a down move "sell signal," the assets in the mutual fund are sold and moved back into the money market for safe keeping until the next up move. The goal is to avoid being invested in mutual funds during a market decline.

Fund of Funds Composite Index. Fund of Funds invest with multiple managers through funds or managed accounts. The strategy designs a diversified portfolio of managers with the objective of significantly lowering the risk (volatility) of investing with an individual manager. The Fund of Funds manager has discretion in choosing which strategies to invest in for the portfolio. A manager may allocate funds to numerous managers within a single strategy, or with numerous managers in multiple strategies. The minimum investment in a Fund of Funds may be lower than an investment in an individual hedge fund or managed account. The investor has the advantage of diversification among managers and styles with significantly less capital than investing with separate managers.

Table 1
Panel A Descriptive Statistics of Volatility Measures

This table presents descriptive statistics on returns and measures of volatility. The sample period is January 1994 to December 2004. The variable CSV is the cross-sectional volatility of stock returns, MV is the time series market volatility. SD is the standard deviation, ρ_1 and ρ_2 are the first-order and second-order autocorrelation respectively.

	Mean	SD	Skewness	Kurtosis	ρ_1	ρ_2
CSV	0.19	0.05	2.31	9.85	0.29	0.26
MV	0.05	0.02	1.21	3.95	0.67	0.57

Panel B Pair-wise Correlation

This table reports pair-wise correlation among the determinants. CSV denotes cross-sectional volatility and MV denotes time-series market volatility. The Fung and Hsieh (2004) factors are S&P 500 return minus risk free rate (SNPMRF), Wilshire small cap minus large cap return (SCMLC), change in the constant maturity yield of the 10-year Treasury (BD10RET), change in the spread of Moody's Baa minus the 10-year Treasury (BAAMTSY), bond PTFS (PTFSBD), currency PTFS (PTFSFX), and commodities PTFS (PTFSCOM), where PTFS denotes primitive trend following strategy.

	CSV	MV	SNPMRF	SCMLC	BD10RET	BAAMTSY	PTFSBD	PTFSFX
CSV	1							
MV	0.30	1						
SNPMRF	0.19	-0.23	1					
SCMLC	0.35	-0.16	-0.09	1				
BD10RET	-0.05	-0.18	0.02	0.07	1			
BAAMTSY	-0.02	0.40	-0.10	-0.27	-0.64	1		
PTFSBD	-0.04	0.27	-0.15	-0.05	-0.10	0.07	1	
PTFSFX	-0.07	0.01	-0.16	0.00	-0.19	0.14	0.15	1
PTFSCOM	-0.14	-0.04	-0.11	-0.01	-0.06	0.06	0.15	0.31

Table 2 Summary statistics of HFR hedge fund indices

This table reports the means, standard deviations, skewness, kurtosis, first order autocorrelation(ρ_1) and minimum and maximum of returns for HFR hedge fund indices

Hedge fund strategy	Mean	Standard deviation	Skewness	Kurtosis	Minimum	Maximum	ρ_1%
Convertible Arbitrage	0.93	0.93	-0.91	5.59	-3.19	3.33	22.3
Distressed Securities	1.07	1.62	-1.81	12.18	-8.50	5.06	23.8
Equity Hedge	1.52	2.69	0.14	4.14	-7.65	10.88	12.7
Equity Market Neutral	0.86	0.94	-0.02	3.25	-1.67	3.59	-0.3
Equity Non-Hedge	1.36	4.27	-0.51	3.49	-13.34	10.74	16.3
Event Driven	1.17	1.93	-1.40	8.44	-8.90	5.13	27.7
Market Timing	0.98	2.05	0.13	2.48	-3.28	5.96	15.8
Fund of Funds Composite	0.71	1.74	-0.29	7.43	-7.47	6.85	12.9

Table 3 Summary statistics of CISDM hedge funds

This table presents cross-sectional means and standard deviations of basic summary statistics for funds in the CISDM database over the sample period January 1993 to August 2004. K is the number of funds. SD denotes standard deviations. $\hat{\rho}_1\%$ and $\hat{\rho}_2\%$ denote first order and second order autocorrelation respectively.

Category	K	Mean		SD		Skewness		Kurtosis		$\hat{\rho}_1\%$		$\hat{\rho}_2\%$	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Mkt neutral	121	0.96	0.57	3.88	2.98	0.44	1.19	6.57	5.70	13.85	17.72	7.54	15.27
Eq hedge	58	0.99	0.84	5.18	2.74	0.01	1.10	6.40	4.85	13.99	16.23	6.88	14.16
Eq nonhedge	20	1.24	0.82	8.25	4.69	0.17	0.69	4.86	1.88	6.97	13.16	-0.01	11.30
Global macro	90	0.96	0.81	5.30	3.54	0.28	0.97	5.60	4.40	9.51	16.81	1.98	15.14
Distressed	72	1.08	0.59	3.82	3.01	-0.14	1.34	7.83	6.36	18.63	16.85	7.81	13.63
Merger arb	106	0.89	0.54	3.04	3.72	-0.17	1.14	6.70	5.20	20.67	15.82	11.63	15.57
Conv. arb	106	1.02	0.51	2.11	1.71	-0.14	1.38	7.18	5.33	30.93	17.31	12.81	16.87
Fixed income	36	0.58	0.35	2.37	1.97	-2.27	2.30	17.10	15.98	19.59	16.53	11.13	21.64

Table 4 Results with hedge fund index

This table reports the results of the regression $a_t = C + \beta_1 CSV_t + \beta_2 MV_t + \epsilon_t$ for the HFR hedge fund indices during the full sample period from January 1994 to December 2004. Where a_t is risk-adjusted hedge fund return index at month t, CSV_t is cross-sectional volatility at month t, MV_t is time series volatility at month t. The t-statistics in parentheses use Newey-West heteroskedasticity and autocorrelation consistent standard errors.

Category	C	CSV	MV	R^2
Convertible arbitrage	0.37 (1.37)	**8.48** **(3.01)**	1.00 (0.14)	0.08
Distressed securities	**1.18** **(3.23)**	−0.42 (−0.23)	−4.90 (−0.62)	0.01
Equity hedge	−0.05 (−0.07)	**8.46** **(1.96)**	**−13.61** **(−1.86)**	0.12
Equity market neutral	**1.01** **(4.77)**	−2.94 (−0.74)	−7.19 (−1.55)	0.04
Equity non-hedge	0.32 (1.04)	**16.23** **(4.03)**	−9.18 (−1.01)	0.12
Event driven	**1.13** **(4.54)**	3.99 (1.14)	−10.53 (−1.78)	0.04
Market timing	−0.35 (−0.57)	5.64 (1.48)	−0.87 (−0.16)	0.05
Fund of funds composite	−0.13 (−0.23)	5.14 (1.44)	−7.57 (−0.82)	0.05

Table 5 Results with pooled regression

This table reports the results of the pooled regression $\hat{a}_{i,t} = b_i + b_1 CSV_t + b_2 MV_t + u_{i,t}$ with panel corrected standard errors (PCSE). Where $\hat{a}_{i,t}$ is the estimated risk-adjusted return of fund i at month t. The parameters b_1 and b_2 are constrained to be the same across funds. Our PCSE specification allows $u_{i,t}$ to be contemporaneously correlated and heteroskedastic across funds, and autocorrelated within each fund's time series. Sample period: January 1994 to August 2004.

	CV	MV
Equity hedge	**12.50** (**4.63**)	**−11.33** (**−2.52**)
Equity non-hedge	**13.48** (**3.02**)	−1.58 (−1.19)
Market neutral	**10.12** (**4.79**)	−1.07 (−0.33)
Merger arbitrage	**3.84** (**1.80**)	**−9.20** (**−2.46**)
Convertible arbitrage	**10.45** (**4.87**)	−0.94 (−0.26)
Distressed securities	1.59 (0.98)	−1.02 (−0.87)

Table 6 Market dispersion and the dispersion in hedge fund returns

This table reports the results of the regression $CSVR_t = \gamma_0 + \gamma_1 CSV_t + \gamma_2 STR_t + \gamma_3 SMB_t + \gamma_4 CSVR_{t-1} + \eta_t$ for three categories of hedge funds during the full sample period from January 1994 to December 2004. Where $CSVR_t$ is cross-sectional dispersion of hedge fund returns at month t, CSV_t is cross-sectional volatility of stock returns at month t, STR_t is stock market return at month t, SMB_t is small minus big portfolio return at month t. The t-statistics in parentheses use Newey-West heteroskedasticity and autocorrelation consistent standard errors.

Category	CSV	STR	SMB	$CSVR_{t-1}$	R^2
Equity hedge	**0.09**	0.03	−0.01	**0.34**	0.21
	(2.21)	(0.66)	(−0.18)	**(3.76)**	
Equity nonhedge	**0.27**	0.00	−0.00	0.00	0.09
	(2.25)	(0.74)	(−0.06)	(1.57)	
Market neutral	**0.08**	0.02	−0.00	**0.28**	0.12
	(1.97)	(0.56)	(−0.11)	**(1.98)**	
Merger arbitrage	**0.12**	−0.04	0.02	**0.21**	0.13
	(2.74)	(−0.80)	(0.50)	**(2.32)**	

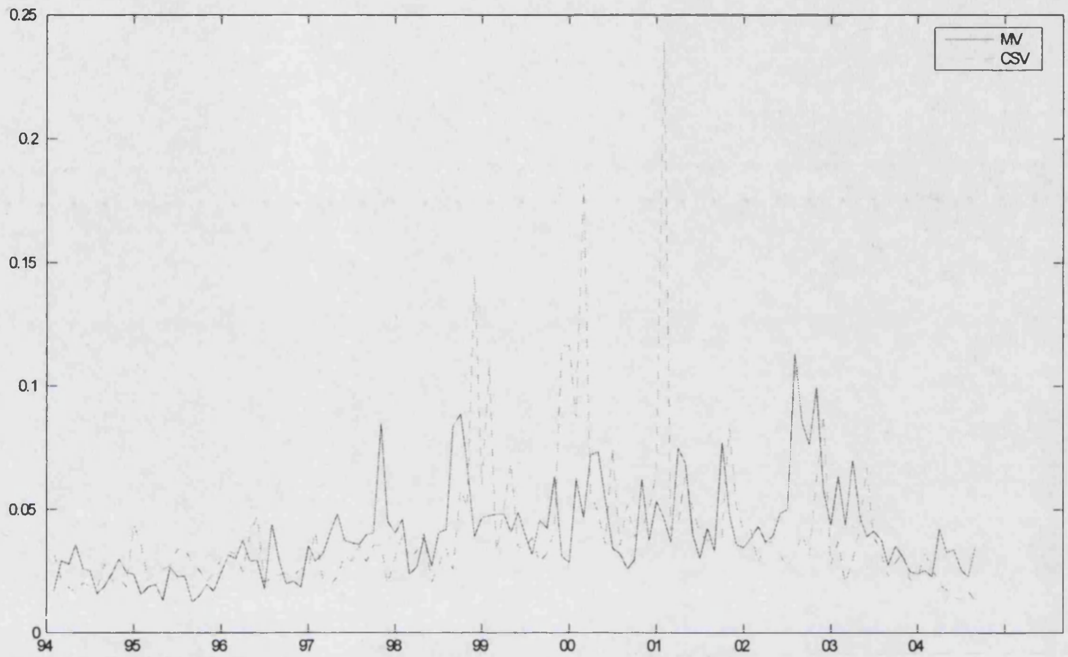

Figure 1 Plot of cross-sectional volatility and time series market volatility(1994-01: 2004-08)

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Chapter 3

An Empirical Comparison of Fung-Hsieh, Fama-French and Statistical Factor Models of Hedge Fund Returns

An Empirical Comparison of Fung-Hsieh, Fama-French and Statistical Factor Models of Hedge Fund Returns*

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September 2007

Abstract

We assess the empirical success of Fung and Hsieh (2004) asset based style factors, a five-factor extension of Fama-French factors and statistical factors for explaining the hedge fund returns. We document that the first two sets of factor models explain a significant part of the systematic exposure of hedge funds and that the explanatory power of the five-factor extension of Fama and French is larger than those of the Fung and Hsieh seven-factor for all categories except for Distressed Securities and Fixed Income Arbitrage. Asymptotic principal component analysis of the individual fund regression residuals reveals that there are latent factors which are not captured by Fung-Hsieh and Fama-French models.

Keywords: Factor models; Hedge fund returns

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1 Introduction

Understanding risk exposure of hedge fund returns is a fundamental issue for hedge fund managers, investors and regulators. A number of factor models have been put forward to explain hedge fund returns. Most evaluations of hedge fund risk depend crucially on these factor models. While some models of hedge fund returns use factors that have been used to explain securities returns, the others develop new factors that are linked to different asset types.

Despite the wide range of models developed, relatively little is known about how these models compare in terms of their ability to capture the actual source of hedge fund returns. There is no well-established method for explaining the dynamic behavior of hedge fund returns in the existing literature. The reason for this is partly due to the large diversity of the strategies employed by hedge funds and their dynamic nature. Previous studies are also based on different data sets, making it more difficult to compare their performance directly. The issue of how these models compare with each other is important since each model differs in its implications for understanding the risk of hedge funds and valuing hedge fund performance.

In this paper, we assess the empirical success of different factor models and compare their explanatory power for the cross section of hedge fund returns. We consider three types of factor models. The first set of models is asset-type based models. We use Fung and Hsieh (2004) seven-factor model (FH hereafter). The seven factors are two equity risk factors (S&P 500, SC-LC), two interest rate risk factors (the change in the yield of the 10 year treasury, and the change in the credit spread), and three trend-following factors (the portfolio returns of options on currencies, commodities, and long term bonds). The second set of models we consider is a five-factor extension of Fama and Frech factors (FF hereafter). The five factors include equity market return, size, value, momentum and cross-sectional volatility factors .

Studying a large cross-section of 609 hedge fund return data over the 1994-2004 period, we document that both FH seven-factor and a five-factor extension of FF explain a significant part of the systematic exposure of hedge funds. The explanatory power of aggregate FH and

FF factors varies across categories, ranging from 17.73% to 54.79% of the total variation in hedge fund returns. We also demonstrate that the explanatory power of FF factors is larger than those of FH seven-factor for all categories except Distressed Securities and Fixed Income Arbitrage, indicating FF factors better capture some of the systematic risk of hedge funds than FH seven factors. This may be partly due to the fact that a large portion of the hedge funds is equity-oriented and FF factors are all equity risk factors. Asymptotic principal component analysis of the individual fund regression residuals reveals that for all categories, FH and FF models successfully capture systematic variation in hedge fund returns. However, the overall explanatory power of the first five factors ranges from 13.89% to 36.34%, indicating there may be some latent factors which have not been captured by previous models.

The remainder of the paper proceeds as follows. Section 2 describes the set of factor models considered here. Section 3 explains the data used in the empirical analysis and discusses the results. Section 4 concludes.

2 Types of factor models

We evaluate three types of factor models in this paper. The first set of models attempt to explain hedge fund returns based on the type of assets traded and the trading style of the hedge fund managers. This approach is adopted from Sharpe's (1992) asset-class factor models. Sharpe decomposes a mutual fund's return into two distinct components: asset-class factors such as large-cap stocks, growth stocks, and intermediate government bonds, which he interprets as "style", and an uncorrelated residual that he interprets as "selection". Following this approach, a number of papers estimate factor models using a broad set of financial and economic factors, including equity index returns, interest rates, exchange rates and commodity prices (Schneeweis and Spurgin, 1998; Liang, 1999; Edwards and Caglayan, 2001; Ennis and Sebastian 2003; Capocci and Hubner, 2004; Hill, Mueller, and Balasubramanian, 2004). To capture the nonlinearities in hedge fund returns due to dynamic trading strategies and derivatives, some also include the returns to certain options-based strategies and other basic portfolios (Fung and Hsieh, 2001, 2004; Agarwal and Naik 2000a,b, 2004). More recently, several papers focus on replicating hedge fund returns. Kat and Palaro (2005, 2006a,b) use sophisticated dynamic trading strategies involving liquid futures contracts to replicate the statistical properties of hedge-fund returns. However, Hasanhodzicy and Lo (2006) argue that some replicating strategies used by Kat and Palaro are too involved, even more complex than the hedge fund strategies they intend to replicate. They estimate linear factor models for individual hedge funds using six common factors, and find that for certain hedge-fund style categories, a significant fraction of funds' expected return can be captured by common factors.

Among these factor models, the risk factors are not necessarily unique. A set of variables may be highly correlated with other variables. Linear combinations of different risk factors can produce substantially similar results. In this paper, we use Fung and Hsieh (2004) seven-factor model because from the practical perspective, their factors are mostly directly linked to conventional asset-class indices given the same level of explanatory power. The seven factors are two equity risk factors (S&P 500, SC-LC), two interest rate risk factors (the change in the yield of the 10 year treasury, and the change in the credit spread), and three trend-following factors (the portfolio returns of options on currencies, commodities, and long term bonds). Fung and Hsieh (2004) show that a large proportion of hedge fund returns can be explained by their factor model.

The second class of models we consider is a five-factor extension of Fama-French three-factor model. The FF model is well-established in the asset pricing literature. We choose this model because it has been successfully applied to returns of stocks, stock portfolios, and mutual funds. The FF model has three factors: a market factor which is the excess return of the market portfolio (MKT hereafter), a size factor which captures return difference between a portfolio of low-capitalization stocks and a portfolio of high-capitalization stocks (SMB hereafter), and a value factor which is the return difference between a portfolio of high book-to-price stocks and a portfolio of low book-to-price stocks (HML hereafter). The fourth factor is the momentum factor as suggested by Jagadeesh and Titman (1993) and Carhart (1997). This captures the difference between the return to a portfolio with high cumulative returns over the past twelve months and the return to a portfolio with low cumulative returns over the past twelve months. The fifth factor is a cross-sectional volatility factor (CSV hereafter). In the second chapter of this thesis, we exploit the information held in the cross-sectional dispersion of equity returns and find that cross-sectional volatility and the performance of hedge funds are positively related across all equity-oriented hedge funds. Containing information very different from other factors, cross-sectional volatility is an important determinant of hedge fund returns. See also Goyal and Santa Clara (2003), Ang, Hodrick, Xing and Zhang (2006a, 2006b), Connor, Hagmann and Linton (2007).

The third class of models is statistical factor models. These models perform maximum likelihood and principal-components-based factor analysis on panel data samples of stock returns to identify the pervasive factors in returns. This approach was applied by Fung and Hsieh (1997a) to hedge funds. They extract five mutually orthogonal principal components of the covariance matrix of their sample of 409 hedge funds and CTAs. Using the hedge funds most highly correlated with these principal components, they construct five "style factors" whose returns are highly correlated to the principal components. Brown and Goetzman (2001) perform cluster analysis to identify hedge fund strategy groups by minimizing the sum of squares within each group. See also Martin (2000), Bares, Gibson and Gyger (2001).

The three types of models are related but have different focuses. There are two factors overlapped between the FH and FF models: equity market return and size factors. While FH covers a broad range of asset types, FF focuses on equity markets. In a statistical factor model, the factors are estimated from the sample returns data by maximizing the fit of the model. Therefore, while statistical factors may yield high in-sample R^2s, they suffer from significant over-fitting bias and also lack economic interpretation. Theoretically, it is possible that a linear combination of the statistical factors is identical to FH and FF factors.

In this paper, we use a linear time-series regression approach to assess how well risk factors identified in previous study explain the variation in hedge fund returns. We first show the explanatory power of each factor model considered separately. Then we test whether the FH and FF models explain all of the pervasive comovements in returns against the alternative that statistical models have pervasive explanatory power not captured by the FH and FF models.

3 Empirical Analysis

3.1 Data

We use monthly returns and accompanying information on both live and "dead" individual hedge funds from January 1994 to August 2004 from the CISDM database. This database provides monthly observations of returns, total net assets, and net asset values, and a fund information file, containing fund name, strategy type, management fees, and other supplementary details. We analyze 8 fund strategies, namely Merger Arbitrage, Distressed Securities, Equity Hedge, Equity NonHedge, Market Neutral, Fixed Income Arbitrage, Convertible Arbitrage, Global Macro. We study funds with at least 48 months of observations, which leaves us with a total of 609 individual hedge funds.

Fung and Hsieh seven factors are from Hsieh's website. S&P 500 return minus risk free rate(S&P), Wilshire small cap minus large cap return (SC-LC), change in the constant maturity yield of the 10-year Treasury (10Y), change in the spread of Moody's Baa minus the 10-year Treasury (Cred Spr), bond PTFS (Bd Opt), currency PTFS (FX Opt), and commodities PTFS (Com Opt), where PTFS denotes primitive trend following strategy.

We obtain the data on MKT, SMB, HML, MOM from Ken French's website. Cross-sectional volatility is constructed using CRSP data from January of 1994 to August of 2004. Each month, we use all the stocks which have a valid return for that month.

Table 1 reports summary statistics on the hedge fund returns over our sample period. For each strategy, the table lists the number of funds and means and standard deviations of basic summary statistics. Panel A in Table 2 contains summary statistics of our risk factors over the sample period. Panel B in Table 2 reports pair-wise correlation among the various factors, and shows that none have excessively high correlation.

3.2 Explanatory power of risk factors

To measure the explanatory power of common risk factors for hedge funds, we first perform a univariate regression for each of the 609 hedge funds in our sample, regressing the hedge fund's monthly returns on individual risk factors. Then we average the estimated coefficients across all funds and report the average values. We repeat the same analysis for each style of funds.

To assess the relative importance of each variable, we present in Table 3 results from univariate regressions of hedge fund returns on individual risk factors. Table 3 shows that hedge fund returns are significantly affected by all risk factors and factor sensitivities vary considerably across categories. Specifically, equity market returns have the strongest positive impact on hedge fund returns both in terms of significance and explanatory power. It is a significant factor across all categories except Fixed Income Funds. Not surprisingly, Equity NonHedge funds has the highest mean beta of 1.01 for equity market return, which explains 40.65% of the variation in hedge fund returns. SMB is the second strongest determinant of hedge fund returns. Similar to equity market return, it is significantly positively related to hedge fund returns in all categories except Fixed Income Funds. It explains 10.53% of the variation in Equity NonHedge funds. In line with the findings of the second chapter, changes in aggregate idiosyncratic volatility have a strong positive impact on returns of Equity Hedge, Equity NonHedge, Equity Neutral, Distressed Securities and Convertible Arbitrage funds. It explains on average 10.39% of the time-series variation in Equity NonHedge fund returns. Next, HML and MOM are significant factors for equity-style funds including Equity Hedge, Equity NonHedge and Equity Neutral. Two interest rate risk, the change in 10-year treasury yields and the change in the yield spread between 10-year treasury and Moody's Baa bonds are significant return drivers in Fixed Income Arbitrage funds. They explain on average 5.84% and 7.00% of the variation respectively. Two interest risk and three trend-following factors are the major risk factors for a small portion of hedge funds.

Next, we analyze the joint explanatory power of all suggested factors in a series of regressions of the type:

$$r_{it} = \alpha_i + \beta'_{1i} S_t + \beta'_{2i} F_t + \varepsilon_{it} \tag{1}$$

where S_t is the vector of FH seven factors realized at time t, F_t is the vector of five-factor extension of FF model at time t, and β'_{1i} and β'_{2i} are the vectors of sensitivities. Table 4 summarizes the results from three different specifications of regression (1), providing a clear picture of the relative power of risk factors in explaining hedge fund returns. We test for the statistical significance of each factor by calculating, for each individual fund regression, the t-statistic for each estimated coefficient, based on Newey-West heteroskedasticity-consistent standard errors. Then for each factor we calculate the percentage of t-statistics that are significant at a 95% confidence level across all funds within the same category. The resulting statistic has an exact binomial distribution under the null hypothesis that the factor sensitivity is zero each period. Table 4 shows the means of the factor sensitivities, the percentage of significant t-statistics for each factor.

Column M1 presents the combined explanatory power of FH seven factors. The explained variation of hedge fund returns range from 9.23% for Fixed Income Arbitrage funds to 44.72% for Equity NonHedge funds. Consistent with the results from univariate regressions, Equity Hedge, Equity NonHedge, Equity Neutral, Merger arbitrage, Distressed Securities and Global Macro funds have strong exposure to the two equity factors: S&P and SC-LC. Distressed Securities also has strong negative exposure to credit spread risk factor and trend-following bond factor. Three trend-following factors, the portfolios of lookback options on bonds, currencies, and commodities are significant return drivers in trend-following funds. They are the major risk factors for 5-10% of hedge funds.

Across eight hedge fund categories, there is a statistically significant intercept term ranging from approximately 55 basis points per month for Fixed Income Arbitrage to 96 basis points for Convertible Arbitrage. There appears to be an average alpha, adjusting for these risk factors.

Column M2 presents the combined explanatory power of a five-factor extension of FF factors: MKT, SMB, HML, MOM and CSV. For Equity Hedge, Equity NonHedge and

Equity Neutral, all five factors are significant. For Merger arbitrage, Distressed Securities and Convertible Arbitrage, five factors except MOM are significant. A small portion of funds in Fixed Income and Global Macro has significant exposure to MKT and CSV. For eight categories except Distressed Securities and Fixed Income, the explanatory power of the five-factor extension is larger than those of FH seven-factor. Specifically, the adjusted R-squared for the FH/FF factors is 22.12%/29.22%, 44.72%/55.53%, 14.60%/24.34%, 21.13%/22.89%, 9.23%/15.81% and 18.56%/19.29% for Equity Hedge, Equity NonHedge, Equity Neutral, Merger Arbitrage, Convertible Arbitrage and Global Macro respectively.

Another significant change is in the intercept term (alpha) of the regressions. Compared with the results of FH seven-factor, both the magnitude and significance level of the intercept term drop dramatically. The average intercepts for Equity Hedge, Equity NonHedge and Global Macro become negative, indicating little to no added value from the average fund manager. These results attest to the importance of five-factor in explaining the source of hedge fund returns.

Next, we include FF factors along with FH seven factors to assess their joint explanatory power (excluding MKT and SMB as they are included in seven factors). Column M3 shows that the magnitudes and significance levels of all remaining variables in the all-inclusive regressions are quantitatively similarly to the previous results. The combination of two sets of factors increase the explanatory power. Adjusted R^2 ranges from 17.73% for Fixed Income Arbitrage to 54.79% for Equity NonHedge.

Finally, while the fund-by-fund results reported above strongly suggest that FH and FF factors capture the systematic exposure of hedge funds, we can use the entire data set to make more definitive statements about the statistical significance of each factor. We report the results of joint (across funds/time) tests of significance in Table 5. In particular, we estimate a fixed fund-specific model of the form

$$r_{it} = \alpha_i + \beta_1' S_t + \beta_2' F_t + \varepsilon_{it} \tag{2}$$

with panel corrected standard errors (PCSE), where now the parameters β_1' and β_2' are constrained to be the same across funds within the same category. Our PCSE specification allows ε_{it} to be contemporaneously correlated and heteroskedastic across funds, and autocorrelated within each fund's time series. The magnitude and significance of individual factors is consistent with previous results but adjusted R^2 decreases.

Overall, the evidence suggests that both the FH seven-factor model and five-factor extension of the FF model capture the systematic exposure of hedge fund returns[1]. In terms of the explanatory power, FF model outperforms FH model for 6 categories, which is consistent with the fact that the equity-style fund is a sizeable portion of the hedge fund industry.

3.3 Statistical factor analysis of hedge fund returns

We further use asymptotic principal components approach, suggested by Connor and Korajczyk (1988), to estimate the pervasive factors influencing hedge fund returns. It is similar to standard principle components except that it relies on asymptotic results as the number of cross-sections grows large and it gives direct estimates of the time-series sample of factor returns, rather than the factor beta matrix. Using an unbalanced panel to extract asymptotic principal components, we first obtain Ω, cross product matrix of returns by defining it element-by-element, each time using only the set of securities which have returns in both of the pair of time dates

$$\Omega_{t\tau} = \frac{1}{n_{t\tau}} \sum_{i=1}^{n_{t\tau}} r_{it} r_{i\tau}, t, \tau = 1, ..., T \tag{3}$$

where $n_{t\tau}$ is the number of assets with returns in both period t and τ. Connor and Korajczyk (1988) find that the first k eigenvectors of Ω can be used as the estimates of the factor returns.

We implement asymptotic principal component (APC hereafter) analysis of (1) hedge

[1]It is well documented that hedge fund returns exhibit substantial serial correlation. Lo (2001) and Getmansky, Lo, and Makarov (2004) have shown that such high serial correlation in hedge-fund returns is likely to be an indication of illiquidity exposure. To remove the impacts of artificial serial correlation on estimates of risk exposure, we adopt the methodology in Getmansky, Lo, and Makarov (2004) to unsmooth hedge fund returns and repeat the above analysis. Overall, the estimated exposure to different factors is largely not affected by smoothing.

fund returns and (2) the residuals of the time-series regressions. These residuals reflect variation unexplained by FH and FF models. Comparing the two illustrates how well FH and FF risk factors capture the systematic exposure of hedge fund returns.

We first estimate the factors by applying asymptotic principal components to the entire sample of hedge funds in each category over the sample period 1994 to 2004. To understand the behavior of the statistical factors in relation to hedge fund returns, we regress hedge fund returns on the first factor, the first five factors, and the first ten factors. We also use the method suggested by Bai and Ng (2002) to select the number of factors[2]. They set up the factor number determination problems as a model selection problem which involves a trade-off between model parsimony and good fit to the data. The tests are variations on Akaike and BIC information criteria-based tests. We repeat the above analysis for the residual of the time-series regressions.

Table 6 summarizes the results of the APC analysis based on hedge fund returns, as well as on the unexplained variation reflected by the regression residuals. For each category, the first common factor captures between 11.55% to 39.78% of hedge fund return variation. The explanatory power of each subsequent principal component is significantly smaller. Not surprisingly, the overall explanatory power of the first five factors is larger than those of FH and FF factors since the statistical factor model is estimated by maximizing explanatory power. Among eight categories, the explanatory power of statistical factors for Equity Neutral is the lowest.

Focusing on the residuals which are from the time-series regressions on the combination of FH and FF factors, the explanatory power of the first principle component declines dramatically, accounting for 1.12% to 13.84% of hedge fund return variation. However, the subsequent principal component does not decline as before. The overall explanatory power of the first five factors ranges from 13.89% to 36.34%. This suggests that there are latent factors which are not captured by FH and FF models.

In summary, our APC analysis provides evidence that FH and FF factors successfully

[2]However, their method suggests a large number of factors. Since our goal is to detect if significant unexplained systematic variation remains in the residuals, rather than establishing the number of factors, we focus on the first five and ten factors.

explain a large part of variation of hedge fund returns but some latent factors may still remain unidentified.

4 Concluding remarks

In this paper, using 609 hedge funds over the 1994-2004 period, we document that both FH seven-factor and a five-factor extension of FF explain a significant part of the systematic exposure of hedge funds. The explanatory power of aggregate FH and FF factors varies across categories, ranging from 17.73% to 54.79% of the total variation in hedge fund returns. We also demonstrate that the explanatory power of FF factors is larger than those of FH seven-factor for all categories except Distressed Securities and Fixed Income Arbitrage, indicating FF factors better capture some of the systematic risk of hedge funds than FH seven factors. This is due to the fact that a large portion of the hedge funds is equity-oriented and FF factors are all equity risk factors. Asymptotic principal component analysis of the individual fund regression residuals reveals that for all categories, FH and FF models successfully capture systematic variation in hedge fund returns. However, the overall explanatory power of the first five factors ranges from 13.89% to 36.34%, indicating there are latent factors which are not captured by the FH and FF models.

Understanding the sources of hedge fund returns is an important area of research. We need a better understanding of this issue while making investment decisions involving hedge funds. A number of factor models have been put forward to explain hedge fund returns. Our results provide useful information on the comparison of explanatory power of these different factor models to investors dealing with portfolio construction and risk management related issue.

Future research may be to develop passive investment approaches that capture latent factors. Moreover, since hedge funds' strategies vary across funds and style categories, to improve the performance of factor models and goodness-of-fit, one could expand the scope of factors and combine different factors to each hedge fund category. More sophisticated nonlinear methods including nonlinear regression, regime-switching processes can be used to capture inherent nonlinearities of certain hedge fund strategies.

Table 1 Summary Statistics of Hedge Funds

This table presents cross-sectional means and standard deviations of basic summary statistics for funds in the CISDM database over the sample period January 1993 to August 2004. K is the number of funds. SD denotes standard deviations.

Category	K	Mean		SD		Skewness		Kurtosis		$\hat{\rho}_1\%$		$\hat{\rho}_2\%$	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Mkt neutral	121	0.96	0.57	3.88	2.98	0.44	1.19	6.57	5.70	13.85	17.72	7.54	15.27
Eq hedge	58	0.99	0.84	5.18	2.74	0.01	1.10	6.40	4.85	13.99	16.23	6.88	14.16
Eq nonhedge	20	1.24	0.82	8.25	4.69	0.17	0.69	4.86	1.88	6.97	13.16	-0.01	11.30
Global macro	90	0.96	0.81	5.30	3.54	0.28	0.97	5.60	4.40	9.51	16.81	1.98	15.14
Distressed	72	1.08	0.59	3.82	3.01	-0.14	1.34	7.83	6.36	18.63	16.85	7.81	13.63
Merger arb	106	0.89	0.54	3.04	3.72	-0.17	1.14	6.70	5.20	20.67	15.82	11.63	15.57
Conv. arb	106	1.02	0.51	2.11	1.71	-0.14	1.38	7.18	5.33	30.93	17.31	12.81	16.87
Fixed income	36	0.58	0.35	2.37	1.97	-2.27	2.30	17.10	15.98	19.59	16.53	11.13	21.64

Table 2

This table contains summary statistics of our risk factors: FH seven-factor: S&P 500 return minus risk free rate(S&P), Wilshire small cap minus large cap return (SC-LC), change in the constant maturity yield of the 10-year Treasury (10Y), change in the spread of Moody's Baa minus the 10-year Treasury (Cred Spr), bond PTFS (Bd Opt), currency PTFS (FX Opt), and commodities PTFS (Com Opt), where PTFS denotes primitive trend following strategy. FF five-factor: the excess return of the market portfolio (MKT), a size factor which captures return difference between a portfolio of low-capitalization stocks and a portfolio of high-capitalization stocks (SMB), and a value factor which captures return difference between a portfolio of high book-to-price stocks and a portfolio of low book-to-price stocks (HML), cross-sectional volatility (CSV).

Panel A: Summary Statistics of Risk Factors

Statistics	Mean	Max	Min	St.Dev.
S&P	0.60	9.31	-14.89	4.44
SC-LC	0.04	16.41	-12.64	3.43
10Y	-0.01	0.65	-0.53	0.24
Cred Spr	0.00	0.448	-0.25	0.13
Bd Opt	0.01	0.66	-0.24	0.16
FX Opt	-0.01	0.90	-0.30	0.19
Com Opt	-0.01	0.65	-0.23	0.13
MKT	0.56	8.18	-16.20	4.56
SMB	0.17	21.87	-16.58	4.23
HML	0.36	13.71	-12.66	3.87
MOM	0.81	18.40	-25.05	5.56
CSV	0.25	23.88	1.31	3.04

Panel B: Correlations

	S&P	SC-LC	10Y	Cred Spr	Bd Opt	FX Opt	Com Opt	MKT	SMB	HML	MOM	CSV
S&P	1.00											
SC-LC	-0.08	1.00										
10Y	0.02	0.05	1.00									
Cred Spr	-0.10	-0.26	-0.64	1.00								
Bd Opt	-0.14	-0.05	-0.09	0.07	1.00							
FX Opt	-0.15	0.01	-0.19	0.14	0.17	1.00						
Com Opt	-0.13	-0.03	-0.03	0.03	0.13	0.29	1.00					
MKT	0.97	0.12	0.03	-0.16	-0.15	-0.14	-0.12	1.00				
SMB	-0.03	0.92	0.09	-0.31	-0.06	0.01	-0.01	0.18	1.00			
HML	-0.43	-0.28	-0.09	0.10	-0.04	0.07	-0.03	-0.54	-0.51	1.00		
MOM	-0.29	0.15	-0.16	0.11	-0.05	0.13	0.18	-0.22	0.17	-0.07	1.00	
CSV	0.17	0.34	-0.04	-0.04	-0.02	-0.05	-0.13	0.25	0.45	-0.43	-0.13	1.00

Table 3: Explanatory Power of Individual Factors

This table reports the results from univariate regressions for each of the 609 hedge funds in our sample over 1994 to 2004, regressing the hedge fund's monthly returns on individual risk factors. Then we average the estimated coefficients across all funds and report the average values

Factor	Equity hedge		Equity non-hedge		Equity neutral		Merger arbitrage		Distressed securities		Convertible arbitrage		Fixed income		Global macro	
	β	R^2	β	R^2	β	R^2	β	R^2	β	R^2	β	R^2	β	R^2	β	R^2
S&P	0.35	18.11	1.01	40.65	0.18	9.68	0.23	14.47	0.31	16.74	0.10	6.04	0.07	4.76	0.33	11.13
SC-LC	0.19	5.25	0.52	10.53	0.14	5.28	0.18	5.11	0.21	7.78	0.08	2.16	0.09	0.19	0.14	4.96
10Y	1.09	0.29	0.86	1.00	0.26	0.09	0.35	-0.10	0.56	1.03	0.24	0.56	1.35	2.84	-0.95	1.17
Cred Spr	-4.65	2.33	-6.34	2.86	-1.18	1.39	-3.01	2.84	-4.73	6.55	-1.75	2.37	-4.95	7.00	-1.44	1.29
Bd Opt	-2.43	1.26	-2.89	0.07	-0.48	0.02	-1.95	2.35	-3.86	4.23	-1.75	1.82	-2.71	3.50	-1.67	1.01
FX Opt	-0.32	-0.47	-4.05	0.00	0.07	0.11	-0.71	-0.32	-0.81	-0.13	-0.38	-0.23	-1.36	1.38	1.34	1.71
Com Opt	-1.82	0.19	-3.91	0.08	-0.87	0.09	-1.78	1.40	-0.68	0.85	-0.63	0.07	-0.97	-0.07	1.56	0.45
HML	-0.23	10.75	-0.85	21.96	-0.13	6.83	-0.17	3.91	-0.19	5.67	-0.09	3.61	-0.03	1.37	-0.23	6.19
MOM	0.00	2.27	-0.10	5.32	-0.02	3.88	-0.04	0.55	-0.02	1.34	-0.01	0.63	0.01	-0.50	0.07	1.65
CSV	0.29	4.93	0.96	10.39	0.24	7.12	0.16	4.42	0.32	4.12	0.22	7.9	-0.08	2.75	0.36	3.22

Table 4 Total Explained Return Variation

This table summarizes the results from three different specifications of regression: $r_{it} = \alpha_i + \beta'_{1i} S_t + \beta'_{2i} F_t + \varepsilon_{it}$, where S_t is the vector of FH seven factors realized at time t, F_t is the vector of five-factor extension of FF model at time t, and β'_{1i} and β'_{2i} are the vectors of sensitivities. Column M1 presents the combined explanatory power of FH seven factors. Column M2 presents the combined explanatory power of a five-factor extension of FF factors. Column M3 reports the joint explanatory power of FF factors and FH seven factors. We report the means of the factor sensitivities. The percentages of significant t-statistics are in parentheses.

Factor	Equity hedge			Equity non-hedge			Equity neutral			Merger arbitrage		
	M1	M2	M3	M1	M2	M3	M1	M2	M3	M1	M2	M3
C	0.79	−0.15	−0.17	0.61	−0.69	−0.77	0.85	0.03	0.03	0.75	0.55	0.57
	(43%)	(2%)	(7%)	(20%)	(20%)	(20%)	(64%)	(17%)	(21%)	(78%)	(37%)	(39%)
S&P	0.33	-	0.38	0.98	-	0.89	0.19	-	0.21	0.22		0.23
	(67%)		(72%)	(100%)		(95%)	(51%)		(53%)	(67%)		(70%)
SC-LC	0.25	-	0.19	0.63	-	0.45	0.21	-	0.15	0.22	-	0.20
	(43%)		(40%)	(80%)		(60%)	(36%)		(38%)	(60%)		(47%)
10Y	−0.04	-	0.49	−1.45	-	−1.69	0.44	-	0.54	−0.60	-	−0.68
	(5%)		(3%)	(5%)		(25%)	(17%)		(19%)	(11%)		(11%)
Cred Spr	−2.12	-	−2.46	−1.09	-	−3.30	2.14	-	1.48	−1.76	-	−2.49
	(17%)		(19%)	(20%)		(20%)	(20%)		(19%)	(22%)		(25%)
Bd Opt	−0.78	-	−0.22	1.46	-	0.90	0.08	-	0.17	−0.84	-	−0.80
	(16%)		(16%)	(15%)		(15%)	(9%)		(9%)	(21%)		(21%)
FX Opt	1.58	-	1.64	0.09	-	0.53	1.06	-	1.04	0.54	-	0.56
	(12%)		(14%)	(10%)		(10%)	(10%)		(10%)	(9%)		(12%)
Com Opt	−0.11	-	−0.08	0.97	-	0.51	−0.22	-	0.48	−0.42	-	0.05
	(5%)		(5%)	(10%)		(0%)	(8%)		(7%0	(16%)		(13%)
MKT	-	0.40		-	0.94		-	0.20		-	0.25	
		(74%)			(100%)			(55%)			(65%)	
SMB	-	0.12		-	0.25		-	0.07		-	0.16	
		(33%)			(35%)			(29%)			(47%)	
HML	-	0.16	0.08	-	0.01	−0.16	-	0.08	0.05	-	0.10	0.01
		(40%)	(34%)		(45%)	(60%)		(33%)	(40%)		(54%)	(38%)
MOM	-	0.06	0.07	-	0.04	0.04	-	0.03	0.03	-	−0.01	−0.02
		(36%)	(41%)		(35%)	(45%)		(32%)	(35%)		(8%)	(17%)
CSV	-	0.16	0.18	-	0.32	0.36	-	0.18	0.18	-	0.02	0.03
		(31%)	(31%)		(32%)	(40%)		(42%)	(40%)		(29%)	(39%)
Adj.R^2	22.12	29.22	30.53	44.72	55.53	54.79	14.60	24.34	25.62	21.13	22.89	24.51

Factor	Distressed securities			Convertible arbitrage			Fixed income			Global macro		
	M1	M2	M3	M1	M2	M3	M1	M2	M3	M1	M2	M3
C	0.90 (74%)	0.58 (42%)	0.55 (54%)	0.96 (85%)	0.27 (33%)	0.24 (33%)	0.55 (61%)	0.77 (39%)	0.53 (33%)	0.57 (46%)	−0.03 (13%)	−0.04 (16%)
S&P	0.29 (65%)	-	0.29 (64%)	0.09 (39%)	-	0.07 (29%)	0.05 (22%)	-	0.07 (19%)	0.33 (57%)	-	0.34 (49%)
SC-LC	0.25 (60%)	-	0.25 (61%)	0.07 (20%)	-	0.02 (17%)	0.06 (11%)	-	0.05 (11%)	0.19 (42%)	-	0.13 (26%)
10Y	−1.37 (19%)	-	−1.41 (24%)	−0.44 (15%)	-	−0.40 (14%)	−0.61 (14%)	-	−0.42 (17%)	−1.75 (23%)	-	−1.39 (13%)
Cred Spr	−5.01 (43%)	-	−5.13 (43%)	−1.84 (22%)	-	−2.00 (26%)	−5.25 (39%)	-	−4.87 (42%)	−3.75 (17%)	-	−3.37 (14%)
Bd Opt	−2.74 (56%)	-	−2.71 (43%)	−1.32 (20%)	-	−1.39 (20%)	−1.59 (17%)	-	−1.49 (28%)	−0.67 (22%)	-	−0.49 (10%)
FX Opt	0.45 (6%)	-	0.48 (4%)	0.17 (3%)	-	0.26 (6%)	−0.67 (6%)	-	−0.69 (14%)	1.94 (18%)	-	1.92 (11%)
Com Opt	0.90 (8%)	-	1.43 (6%)	0.07 (9%)	-	0.60 (8%)	0.28 (14%)	-	0.39 (0%)	1.63 (13%)	-	1.44 (13%)
MKT	-	0.35 (72%)		-	0.09 (35%)		-	0.11 (22%)		-	0.36 (58%)	
SMB	-	0.24 (69%)		-	0.04 (18%)		-	0.11 (22%)		-	0.09 (24%)	
HML	-	0.19 (54%)	0.05 (32%)	-	0.05 (19%)	0.02 (20%)	-	0.09 (17%)	0.02 (8%)	-	0.10 (29%)	0.02 (23%)
MOM	-	−0.01 (18%)	−0.03 (21%)	-	0.01 (17%)	0.00 (16%)	-	0.06 (8%)	0.00 (11%)	-	0.08 (29%)	0.06 (21%)
CSV	-	0.03 (21%)	0.09 (22%)	-	0.14 (54%)	0.17 (53%)	-	−0.16 (!9%)	−0.03 (18%)	-	0.11 (26%)	0.14 (20%)
Adj.R^2	28.41	27.35	31.78	9.23	15.81	17.78	14.58	8.59	17.73	18.56	19.29	22.83

Table 5 FH and FF Factors Across Styles Pooled regressions

This table reports the results of joint (across funds/time) tests of significance. We estimate a fixed fund-specific model: $r_{it} = \alpha_i + \beta_1' S_t + \beta_2' F_t + \varepsilon_{it}$, with panel corrected standard errors, where the parameters $\beta_1'$and β_2' are constrained to be the same across funds within the same category. Our PCSE specification allows ε_{it} to be contemporaneously correlated and heteroskedastic across funds, and autocorrelated within each fund's time series.

Factor	Equity hedge		Equity non-hedge		Equity neutral		Merger arbitrage		Distressed securities		Convertible arbitrage		Fixed income		Global macro	
	β	t	β	t	β	t	β	t	β	t	β	t	β	t	β	t
C	-0.15	-0.60	-0.68	-1.74	0.07	0.50	0.52	3.54	0.61	3.14	0.98	3.03	0.37	2.59	0.03	0.15
S&P	0.37	10.91	0.92	17.94	0.19	9.63	0.23	11.01	0.24	8.97	0.08	4.56	0.03	1.45	0.29	10.02
SC-LC	0.21	5.12	0.44	7.26	0.18	7.17	0.21	8.31	0.21	6.37	0.05	2.20	0.01	0.59	0.15	4.29
10Y	0.37	0.53	-0.94	-0.89	0.63	1.56	-0.60	-1.46	-1.51	-2.79	-0.58	-1.41	-0.23	-0.54	-1.75	-1.86
Cred Spr	-2.35	-1.81	-3.00	-1.56	1.55	1.10	-1.23	-1.62	-4.64	-4.65	-1.66	-2.15	-3.43	-4.38	-3.82	-3.54
Bd Opt	-0.60	-0.75	0.86	0.72	0.16	0.34	-1.03	-2.15	-3.25	-5.16	-1.34	-2.81	-1.88	-3.73	-0.87	-1.23
FX Opt	1.53	1.16	0.72	0.68	1.17	2.68	0.41	0.94	0.37	0.64	0.25	0.57	-0.66	-1.44	1.89	1.90
Com Opt	0.19	0.19	0.80	0.51	0.40	0.64	-0.05	-0.08	1.03	1.25	-0.02	-0.03	0.25	0.39	1.78	0.97
HML	0.11	2.55	-0.20	-3.27	0.05	2.16	0.01	0.51	0.03	1.10	0.03	1.43	0.03	1.22	0.03	0.84
MOM	0.08	3.46	0.07	2.03	0.02	1.72	0.00	0.25	-0.01	-0.46	0.01	0.53	0.02	1.70	0.10	4.96
CSV	0.18	3.96	0.31	4.44	0.18	6.53	0.05	1.79	0.07	1.17	0.14	4.27	0.42	1.58	0.14	1.62
Adj.R^2	11.85		32.82		5.77		7.67		12.50		7.41		5.22		7.48	

Table 6. Asymptotic Principal Component Analysis

We report the explanatory power (adjusted R^2) of statistical factors. We implement asymptotic principal component analysis of (1) hedge fund returns and (2) the residuals of the time-series regressions.

Factor numbers	Equity hedge	Equity non-hedge	Equity neutral	Merger arbitrage	Distressed securities	Convertible arbitrage	Fixed income	Global macro
Returns								
1	23.20	39.78	13.80	11.55	25.34	20.24	22.16	15.93
5	45.55	69.28	28.97	32.31	43.68	40.91	45.84	29.60
10	57.30	81.58	37.82	46.92	57.22	51.46	69.29	39.73
Residuals								
1	6.75	5.63	5.97	1.12	6.91	5.23	13.84	8.18
5	27.38	36.80	13.89	19.01	24.41	30.37	36.34	16.76
10	43.63	60.83	25.95	32.09	37.22	41.71	59.67	30.42

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Chapter 4

Evaluating Hedge Fund Performance: a Stochastic Dominance Approach

Evaluating Hedge Fund Performance: a Stochastic Dominance Approach

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Abstract

We introduce a general and flexible framework for hedge fund performance evaluation and asset allocation: stochastic dominance (SD) theory. Our approach utilizes statistical tests for stochastic dominance to compare the returns of hedge funds. We form hedge fund portfolios by using SD criteria and examine the out-of-sample performance of these hedge fund portfolios. Compared to performance of portfolios of randomly selected hedge funds and mean-variance efficient hedge funds, our results show that fund selection method based on SD criteria greatly improves the performance of hedge fund portfolio.

Keywords: Alpha; Mean Variance analysis; Portfolio; Risk Return

1 Introduction

Over the last decade, the number of hedge funds has risen by about 20 percent per year to reach around 8,500 in 2006. The amount of assets under management of the hedge fund

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industry has increased from around $40 billion in 1990 to an estimated $1,400 billion in 2006. Since hedge funds typically use leverage, the positions that they take in the financial markets are large enough to move markets around the world. The rapid growth in hedge funds reflects the increasing importance of this alternative investment category for institutional investors and wealthy individual investors.

Correspondingly, identifying hedge fund managers with superior skills and refining the traditional portfolio management tools to optimize investments in a large universe of hedge funds have also become challenging tasks in portfolio management. If the top hedge fund performance can be explained by superior skills owned by managers not by luck, we would expect top performance of such managers persists. However, there is little consensus on hedge fund performance persistence in the empirical finance literature. A number of studies find that hedge fund performance only persist at short term (one to three months) which might be due to hedge funds' illiquid exposure and there is no evidence of performance persistence at annual horizons.(see Getmansky, Lo, and Makarov, 2004, Brown, Goetzmann and Ibbotson, 1999, Agarwal and Naik, 2000, Liang, 2000, Bares, Gibson and Gyger, 2003, Boyson and Cooper, 2004, Baquero, Ter Horst and Verbeek, 2005). On the contrary, more recent study by Kosowskia, Naik and Teo (2006) finds that sorting hedge funds on Bayesian alphas yields a 5.5 percent per year increase in the alpha of the spread between the top and bottom hedge fund deciles. Hedge fund performance persists at annual horizon. Using a novel GMM procedure to estimate alpha for hedge fund managers, Jagannathan, Malakhov and Novikov (2006) find evidence of hedge fund managers' performance persistence over three year horizons.

More practical issue facing hedge fund investors is how to construct an efficient hedge fund portfolio or add hedge funds to the existing portfolio. The standard mean–variance approach to portfolio allocation, which is founded on the assumption of normal distributions and an objective function of maximizing risk-adjusted return, is inadequate when dealing with portfolios of hedge funds. A number of studies (see Lo, 2001, Amin and Kat, 2003) have shown that risk characteristics of hedge funds are substantially different from those of traditional investment pools because hedge fund managers usually employ highly dynamic trading strategy and use short selling, leverage, concentrated investments, and derivatives. Specifically, hedge fund returns are not normally distributed and exhibit significant skewness and kurtosis. They also tend to display significant co-skewness with the returns on other hedge funds as well as equity. Mean-variance models ignore these higher moments of the return distribution, and thus fail to take into consideration the benefits of funds that occasionally surprise on the upside while they also underestimate the risk of funds that have asymmetric downside risk. Despite the weakness of mean-variance frame work, it still dominates in practical hedge fund portfolio management. The Sharpe ratio is commonly used to quantify the risk-return trade-off. Amenc, Giraud, Martellini and Vaissie (2004) report that only 2% of the European multi-managers pay attention to skewness and kurtosis; while 84% of multi-manager funds consider that volatility is of major concern to their clients and 82% consider Sharpe ratio as an important indicator. A number of studies also address the issue of including hedge funds in standard institutional portfolios in mean-variance portfolio optimization. (see, Amenc and Martellini, 2002, Brunel, 2004, Kat, 2005, Till, 2005).

Another strand of literature develops different frameworks for hedge fund allocation, which incorporate a variety of investment objectives, particularly investor preferences for skewness and kurtosis of returns, into portfolio optimization models. Using a Polynomial Goal Programming (PGP) optimization model, Davies, Kat and Lu (2005) solve for multiple competing hedge fund allocation objectives within a mean-variance-skewness-kurtosis framework and analyze different impacts of various hedge fund strategies on the distribution of optimal portfolio. Morton, Popova and Popova (2006) study hedge fund allocation issue by assuming a family of utility functions which are a weighted sum of the probability of achieving a benchmark and expected regret relative to another benchmark. They then use a Monte Carlo method to obtain a solution to the related portfolio optimization model. Alexander and Dimitriu (2004) develop a portfolio construction model by selecting funds according to their ranking of alpha estimated with factor models. They then allocate selected funds using constrained minimum variance optimization.

In this paper, we introduce a more general and flexible framework for hedge fund asset allocation — stochastic dominance (SD) theory. Our approach utilizes statistical tests for stochastic dominance to compare the returns of hedge funds. The theory of stochastic dominance (see, Hadar and Russell, 1969, Hanoch and Levy, 1969, Rothschild and Stiglitz, 1970, and Whitmore, 1970) provides a systematic framework for analyzing economic behavior under uncertainty. We form hedge fund portfolios by using SD criteria. We then examine the out-of-sample performance of these hedge fund portfolios. Compared to both randomly selected hedge fund portfolio and mean-variance efficient hedge fund portfolio, our results show that fund selection method based on SD criteria greatly improves the performance of hedge fund portfolio.

Our framework relying on stochastic dominance has several advantages. First, we are able to use the information embedded in the entire empirical return distributions of hedge funds instead of a finite set of sample statistics. Second, while mean-variance analysis is consistent with the expected utility theory only under relatively restrictive assumptions about investor preferences or the statistical distribution of the investment returns, SD criteria do not require a full parametric specification of investor preferences, but rather rely on general preference assumptions which are intuitively close to the real objectives of investors, for example, non-satiation in the case of first order stochastic dominance (FSD) and risk aversion in the case of second order stochastic dominance (SSD). This is important because the view of investors towards various hedge funds depends crucially on their investment objectives and risk preferences.

The reminder of the paper is organized as follows. Section 2 introduces stochastic dominance framework. Section 3 describes the data and reports the results of empirical analysis and a comparison of performance of various hedge fund portfolios constructed by using different criteria. Section 4 concludes.

2 Stochastic Dominance

Stochastic dominance theory provides a possible comparison relationship between two stochastic distributions. Stochastic dominance relations offer a general decision rule for decision making when facing the choice between random payoffs, given that the utility functions share some common characteristics such as non-satiation or risk-aversion. In this paper, we test for the first and second orders of stochastic dominance.

Let X_1 and X_2 be two outcome variables. Let $\mathcal{U}_1$ denote the class of all von Neumann-Morgenstern type utility functions, u, such that $u' \geq 0$, (increasing). Also, let $\mathcal{U}_2$ denote the class of all utility functions in $\mathcal{U}_1$ for which $u'' \leq 0$ (strict concavity). Let $F_1(x)$ and $F_2(x)$ denote the cumulative distribution functions, respectively.

Definition 1 *X_1 First Order Stochastic Dominates X_2, denoted $X_1 \succeq_{FSD} X_2$, if and only if:*
(1) $E[u(X_1)] \geq E[u(X_2)]$ for all $u \in \mathcal{U}_1$, with strict inequality for some u; Or
(2) $F_1(x) \leq F_2(x)$ for all x with strict inequality for some x.

Definition 2 *X_1 Second Order Stochastic Dominates X_2, denoted $X_1 \succeq_{SSD} X_2$, if and only if either:*
(1) $E[u(X_1)] \geq E[u(X_2)]$ for all $u \in \mathcal{U}_2$, with strict inequality for some u; Or:
(2) $\int_{-\infty}^{x} F_1(t)dt \leq \int_{-\infty}^{x} F_2(t)dt$ for all x with strict inequality for some x.

For any two outcomes i, j define

$$\delta_{ij} = \sup_{x \in \mathcal{X}} F_i(x) - F_j(x),$$

where $\mathcal{X}$ is contained in the supports of X_i, X_j. Fund i dominates fund j if $\delta_{ij} \leq 0$. If a fund X_1 second order dominates fund X_2 then no risk averse individual would prefer X_2 to X_1. First order dominance of one outcome by another is even stronger: If a fund X_1 first order dominates fund X_2 then no individual who prefers more wealth to less would prefer X_2 to X_1. First order dominance implies second order dominance. Note that these concepts do not require the existence of moments of the underlying outcomes unlike mean variance analysis. Furthermore, both relations are transitive, i.e., if X_1 dominates X_2 and X_2 dominates X_3 then X_1 dominates X_3. However, neither relation denotes a full ordering, only a partial ordering. That is, we may not be able to rank two outcomes at all according to either relation. In such cases, one can either say one is indifferent between the two investments or one can impose more preference structure to discriminate between them. One possibility is to increase the dominance order to third order or fourth order etc., which reduces the set of noncomparability. Alternatively one can then supplement the partition induced by the dominance relation by some additional criterion like Sharpe ratio. In practice, although FSD implies strong relationship between two outcomes, it is not very discerning because the cumulative distributions of net returns of the two investment alternatives often intersect, in which case FSD cannot discriminate between the alternatives. For decision making under risk more important is SSD. If investors are risk averse and prefer more to less, SSD could be used to choose between two outcomes.

In empirical analysis, stochastic dominance analysis requires the comparison of the probability distributions of two outcomes which are unknown and must be estimated from available data. Various statistical tests for the existence of SD orders have been developed. Several tests proposed earlier (for example Anderson, 1996 and Davidson and Duclos, 2000) compare the distribution functions only at a fixed number of arbitrarily chosen points. In general, comparisons using only a small number of arbitrarily chosen points will have low power if there is a violation of the inequality in the null hypothesis on some subinterval lying between the evaluation points used in the test. More recent tests proposed by Barrett and Donald (2003) and Linton, Maasoumi and Whang (2003) compare the two distributions at all points in the sample.

3 Empirical Results

3.1 Description of the data

In this section, we provide an empirical analysis of hedge fund database under Stochastic Dominance framework. The database used in this paper covers the period January 1994 to August 2004 and was provided by the Center for International Securities and Derivatives Markets (CISDM). It has two parts: a total of 1,269 live hedge funds and 1,760 dead hedge funds. To reduce survivorship bias, we include both live and dead funds in our analysis.

Each set consists of a performance file, containing monthly net-of-fee returns, total net assets, and net asset values, and a fund information file, containing fund name, strategy type, management fees, and other supplementary details. We select only those funds with at least 2 years of monthly observations. To ensure there are enough funds in different fund categroies, we analyze 6 fund strategies, namely Merger Arbitrage, Distressed Securities, Equity Hedge, Market Neutral, Convertible Arbitrage and Global Macro. Table 1 lists summary statistics of the hedge funds from CISDM database during the January 1994 to August 2004 period. For each strategy, the table lists the number of funds and means and standard deviations of basic summary statistics.

3.2 Results

To compare hedge fund returns using stochastic dominance concepts, our procedure includes two steps. First, we take into account the systematic risk exposure of hedge funds and obtain the risk-adjusted returns of hedge funds. Then we test for FSD and SSD relations among risk-adjusted hedge fund returns relying on Linton, Maasoumi and Whang (2003) statistical test.

Risk adjustments for hedge fund returns are difficult due to their use of derivatives and dynamic trading strategies. Commonly used methods include using hedge fund indices and factor models. More recently, a number of studies (see Kat and Palaro (2005)) argue that sophisticated dynamic trading strategies involving liquid futures contracts can replicate many of the statistical properties of hedge-fund returns. Hasanhodzic and Lo (2006) estimate linear factor models for individual hedge funds using six common factors, and find that for certain hedge-fund style categories, a significant fraction of funds' expected return can be captured by common factors.

Here we use as performance benchmarks the seven-factor model developed by Fung and Hsieh (2004). The Fung and Hsieh (2004) factors are S&P 500 return minus risk free rate (SNPMRF), Wilshire small cap minus large cap return (SCMLC), change in the constant maturity yield of the 10-year Treasury (BD10RET), change in the spread of Moody's Baa minus the 10-year Treasury (BAAMTSY), bond PTFS (PTFSBD), currency PTFS (PTFSFX), and commodities PTFS (PTFSCOM), where PTFS denotes primitive trend following strategy. Fung and Hsieh (2004) show that their factor model strongly explains variation in individual hedge fund returns.

In order to obtain risk-adjusted performance of hedge funds, we regress the net-of-fee monthly excess return (in excess of the risk free rate) of a hedge fund on the seven-factor model.

$$R_{i,t} = \alpha_i + \beta_i^{\top} Z_t + \epsilon_{i,t}, \tag{1}$$

where β_i represents the risk exposure of fund i at month t to the various factors, and Z_t is the monthly value of different factors. The risk-adjusted return of fund i at month t is calculated as:

$$\hat{a}_{i,t} = R_{i,t} - \hat{\beta}_i^{\top} Z_t = \hat{\alpha}_i + \hat{\epsilon}_{i,t}, \tag{2}$$

where $R_{i,t}$ is the net-of-fee monthly excess return of fund i in month t, $\hat{\beta}_i$ is the estimated risk exposure for fund i, and Z_t is the value of the various factors at month t. We compute the risk-adjusted returns $\hat{a}_{i,t}$ as the sum of the intercept $\hat{\alpha}_i$ and the residual $\hat{\epsilon}_{i,t}$ of Eq.(1). We plot the distribution of the intercept α in Figure 1.

We next conduct an analysis of the distributions of risk-adjusted returns of the funds, with a view to establishing stochastic dominance orderings. For each fund i we compute the empirical c.d.f. and integrated c.d.f. [denoted s.d.f.] as follows

$$\begin{aligned}
\widehat{F}_i(x) &= \frac{1}{T_i}\sum_{t=1}^{T_i} 1(X_{it} \leq x) \\
\widehat{S}_i(x) &= \int_{-\infty}^{x} \widehat{F}_i(x')dx' = \frac{1}{T_i}\sum_{t=1}^{T_i}(x - X_{it})1(X_{it} \leq x),
\end{aligned}$$

where $X_{it} = \hat{a}_{i,t}$ is risk-adjusted return. We say that a fund i is first order dominated if for some fund j

$$\max_{1 \leq \ell \leq L} \widehat{F}_j(x_\ell) - \widehat{F}_i(x_\ell) < 0,$$

where $x_1, \ldots, x_L$ is a grid of points contained in the union of the supports of the distributions. Likewise, for second order dominance.

Let $\mathcal{F}_D = \{i : i \text{ is first order dominated}\}$ and let $\mathcal{F}_U$ be the complement of this set in the full set of funds, likewise define $\mathcal{S}_D$ and $\mathcal{S}_U$. Clearly, $\mathcal{F}_D \subseteq \mathcal{S}_D$ and so $\mathcal{S}_U \subseteq \mathcal{F}_U$.

We compute the set of all funds that are undominated across all pairwise comparisons. We then construct a portfolio of all undominated funds. To examine the out-of-sample performance of undominated funds, we construct portfolios of funds $\mathcal{S}_U$ on January 1 each year (from 1999 to 2004), based on stochastic dominance orders of risk-adjusted hedge fund returns estimated over the prior five years. The portfolios are equally weighted monthly, so the weights are readjusted whenever a fund disappears[1]. We also construct the portfolio of first-order dominated funds for comparison purpose. Given the economic intuition of stochastic dominance that any risk averse individual should choose funds in $\mathcal{S}_U$ and any investor who prefer more to less should not choose funds in $\mathcal{F}_D$, we expect portfolio of funds in $\mathcal{S}_U$ exhibit much better performance than portfolio of funds in $\mathcal{F}_D$.

To compare stochastic dominance tests with mean variance tests, we also apply mean and variance efficient criteria to risk-adjusted returns of hedge funds. We construct portfolios of mean-variance efficient funds on January 1 each year (from 1999 to 2004), based on means and variances of risk-adjusted returns of funds estimated over the prior five years. A fund is defined as a mean-variance efficient fund if no other funds have both higher means and lower variances than this fund. Hence, funds are selected by comparing only two summary statistics: the mean and the variance, which represents the distribution of risk-adjusted hedge fund returns.

A number of studies find that hedge fund portfolio return properties vary substantially with the number of hedge funds included in the portfolio. See, Amin and Kat (2002), Davies,

[1]Under a pessimistic scenario, the money invested into disappeared hedge funds cannot be recovered. Hence we assume -100% return to a fund during the month after it disappears from the database, and zero returns thereafter.

Kat and Lu (2003), Alexander and Dimitriu (2004). A hedge fund portfolio including only ten funds will typically have significantly higher variance than a similar hedge fund portfolio containing 100 funds. Therefore, to assess the robustness of SD analysis, we also construct representative portfolios containing the same number of funds as in $\mathcal{S}_U$ for each year (from 1999 to 2004). The funds in portfolios are randomly selected.

Table 2 reports the number of hedge funds held by portfolios for each year and Table 3 reports summary statistics and alphas of portfolios constructed using different criteria. Alpha is estimated using the seven-factor model. As we can see in table 2, the number of funds in $\mathcal{S}_U$ is around 30 which is close to those of mean-variance efficient funds while the number of funds in $\mathcal{F}_D$ is substantially larger, ranging from 849 to 1225. French, Ko and Abuaf (2005) examine the current fund of hedge funds universe, and find that funds of hedge funds report holding between 1 and 200 underlying funds, and generally hold 10-30, with close to 20 on average. Hence, the number of holdings in $\mathcal{S}_U$ and mean-variance efficient sets is actually close to practitioner standards. Amin and Kat (2003) also find that the optimal size of well diversified hedge fund portfolios is in the range of 15 to 20.

According to Table 3, the mean return of portfolio of funds in $\mathcal{S}_U$ is 0.99 which is substantially larger than those of other portfolios. The first two moments of returns provide a great deal of the information about the investment outcome set of portfolios, but not everything. We find the skewness for portfolio of funds in $\mathcal{S}_U$ is 2.36 which is much larger than for other portfolios. Positive skewness means essentially that the big outcomes are on the upside so there is relatively little chance of large negatives. From a variety of points of view positive skewness is desirable.

Moreover, the portfolio of funds in $\mathcal{S}_U$ generates an alpha of 9.91 percent per year. As the t-statistics in column ten shows this alpha is statistically significant. The alphas of portfolios of funds in $\mathcal{F}_D$ and mean-variance efficient funds are also statistically significant but much lower than the alpha of portfolio of funds in $\mathcal{S}_U$. The alpha of the randomly picked funds portfolio is the lowest and statistically insignificant.

Figure 2,3,4 plot the time series of returns of these representative portfolios. We also plot the cumulative returns[2] of representative portfolios and Standard Poor's 500 index in Figure 5. As we can see from the figure, the portfolio constructed by using SD criterion achieves a much higher cumulative return than those of other portfolios.

To further investigate the nature of the stochastic dominance approach, we establish stochastic orders within each style category. We then repeat the above performance analysis. Table 4 reports the results for each category. We find that overall the portfolio of funds in $\mathcal{S}_U$ display superior performance in all categories. In particular, the portfolio of funds in $\mathcal{S}_U$ in the Merger arbitrage category achieve relative higher alpha than that of mean variance efficient funds. For equity neutral funds, the performance of stochastic dominance approach is not better than that of mean-variance approach.

It is also well documented that hedge fund returns exhibit substantial serial correlation, see Getmansky, Lo, and Makarov (2004) and Okunev and White (2003). They suggest that hedge funds' exposure to illiquid assets is the primary source of the strong observed serial correlation in hedge fund returns. To remove the effects of artificial serial correlation, we employ the methodology in Getmansky, Lo, and Makarov (2004) to unsmooth hedge fund returns and repeat the above analysis on the unsmoothed hedge fund returns. Overall, the performance of stochastic dominance approach still dominates other approaches.

4 Concluding remarks

In this paper, we introduce a general and flexible framework for hedge fund performance evaluation and asset allocation. Our approach utilizes recent advances in statistical tests for stochastic dominance. The approach is able to recognize and use the information embedded in the non-normal return distributions of hedge funds. To illustrate the method's ability to work with non-normal distributions, we form hedge fund portfolios by using SD criteria and examine the out-of-sample performance of these hedge fund portfolios. Compared to performance of portfolios of randomly selected hedge funds and mean-variance efficient hedge

[2]The cumulative return is the compound return of the series: $CumR_n = (\prod_{i=1}^{n}(1+r_i)) - 1$, where $r_1, .., r_n$ is a monthly return series.

funds, our results show that fund selection method based on SD criteria greatly improves the performance of hedge fund portfolio. The mean return of portfolio of funds in S_U is substantially larger than those of other portfolios. We also find that the skewness for portfolio of funds in S_U is 2.36 which is much larger than for other portfolios. Positive skewness is desirable because it means essentially that the big outcomes are on the upside so there is relatively little chance of large negatives. Mean-variance optimization models do not necessary achieve this result. Different specifications of investor preferences will result in considerable differences in the impact of skewness on optimal hedge fund allocations.[3]

There are a number of potential areas for improvement. First, the equal weighting of undominated funds can be replaced by more targeted weighting based on some univariate performance criterion like Sharpe ratio. Second, we could look at higher order dominance or asymmetric dominance notions like Prospect or Markowitz dominance. Third, we could take account of sampling variation in constructing the set of undominated funds by including those funds that are within some distance (controlled according to a statistical criterion like Type 1 error using the results of Linton, Maasoumi, and Whang (2003)) from being dominated. This would enlarge the set of undominated funds and it may not improve performance out of sample. Finally, although our SD test shows ability in distinguishing good funds from bad funds, it is restricted to pairwise comparison of a finite number of choice alternatives, and it has limitations with full diversification possibilities. The problem is that the ordering of the outcomes of a diversified portfolio of funds cannot be determined in a straightforward way from the orderings of the individual funds. Therefore, the ordering of each portfolio has to be determined individually. A number of recent studies recently developed Linear Programming (LP) tests for SD that do fully account for diversification.[4] Post and Versijp (2006) develop tests for SSD and TSD efficiency that are embedded in the Generalized Method of Moments (GMM) framework. This test has superior statistical properties to the above LP tests and is a serious rival to the dominant mean-variance tests. We leave the application of these tests to hedge funds as future research.

[3]See Brockett and Kahane (1992), Cremers, Kritzman, and Page (2005)

[4]See Post (2003), Kuosmanen (2004), Post and Levy (2005) and Post and van Vliet (2006)

Table 1
Summary statistics of hedge fund returns

This table presents means and standard deviations of basic summary statistics for funds in the CIDSM database over the sample period January 1994 to August 2004. SD denotes standard deviations. $\hat{\rho}_1\%$ and $\hat{\rho}_2\%$ denote first order and second order autocorrelation respectively.

Category	Mean		SD		Skewness		Kurtosis		$\hat{\rho}_1\%$		$\hat{\rho}_2\%$	
	Mean	SD	Mean	SD	Mean	SD	Mean	S D	Mean	SD	Mean	SD
Market neutral	0.96	0.57	3.88	2.98	0.44	1.19	6.57	5.70	13.85	17.72	7.54	15.27
Equity hedge	0.99	0.84	5.18	2.74	0.01	1.10	6.40	4.85	13.99	16.23	6.88	14.16
Distressed securities	1.08	0.59	3.82	3.01	-0.14	1.34	7.83	6.36	18.63	16.85	7.81	13.63
Merger arbitrage	0.89	0.54	3.04	3.72	-0.17	1.14	6.70	5.20	20.67	15.82	11.63	15.57
Convertible arbitrage	1.02	0.51	2.11	1.71	-0.14	1.38	7.18	5.33	30.93	17.31	12.81	16.87
Global Macro	0.96	0.81	5.30	3.54	0.28	0.97	5.60	4.40	9.51	16.81	1.98	15.14

Table 2
Number of funds

This table reports the numbers of funds held in portfolios of second order undominated funds (S_U), mean-variance efficient funds, first order dominated funds (F_D) over the sample period: January 1999 to August 2004. The portfolios are constructed on January 1 each year. N is the number of funds.

Year	Total N of funds	N of funds in F_D	N of funds in S_U	N of MV efficient funds
1999	1054	849	13	27
2000	1269	947	20	30
2001	1405	1074	22	25
2002	1532	1046	37	24
2003	1582	1084	30	31
2004	1639	1225	19	28

Table 3
Summary statistics of returns for representative portfolios

This table reports summary statistics of returns for portfolios of second order undominated funds (S_U), mean-variance efficient funds, first order dominated funds (F_D), randomly selected funds over the sample period: January 1999 to August 2004. The portfolios are constructed on January 1 each year.

	Mean	Median	Max	Min	Std.Dev.	Skew	Kurtosis	Alpha (pct/year)	t-stat of alph
funds in S_U	0.99	0.83	13.09	-3.25	2.42	2.36	11.98	9.91	3.25
MV efficient funds	0.69	0.75	7.82	-2.78	1.61	1.41	8.25	7.01	3.81
funds in F_D	0.48	0.50	6.07	-3.98	2.04	0.04	2.91	4.42	3.66
funds randomly picked	0.32	0.01	8.08	-4.96	2.30	0.55	4.13	2.19	1.12

Table 4
Summary statistics of returns for representative portfolios within styles

For each style category, this table reports reports summary statistics of returns for portfolios of second order undominated funds (S_U), mean-variance efficient funds, randomly selected funds over the sample period: January 1999 to August 2004. The portfolios are constructed on January 1 each year.

	Mean	Median	Max	Min	Std.Dev.	Skew	Kurtosis	Alpha (pct/year)	t-stat of alph
Equity hedge									
funds in S_U	1.28	0.35	27.78	-4.49	4.43	3.57	20.59	0.89	1.96
MV efficient funds	1.41	0.53	18.82	-3.24	3.88	1.82	9.57	1.07	2.54
funds randomly picked	0.9	0.62	12.01	-5.05	4.99	1.03	8.91	0.86	1.85
Equity neutral									
funds in S_U	0.94	0.75	11.91	-2.49	1.84	3.11	19.81	0.81	4.36
MV efficient funds	0.95	0.86	8.86	-2.89	1.54	2.13	13.04	0.85	5.13
funds randomly picked	0.71	0.49	5.54	-3.58	1.75	0.19	3.62	0.64	3.82
Merger arbitrage									
funds in S_U	0.81	0.8	6.77	-3.05	1.59	0.74	5.75	0.71	4.38
MV efficient funds	0.61	0.46	4.08	-1.75	0.94	0.95	7.81	0.62	4.78
funds randomly picked	0.53	0.52	10.54	-6.45	2.71	0.54	5.41	0.29	1.22
Distressed securities									
funds in S_U	1.05	0.96	4.05	-2.68	1.39	-0.15	3.37	1.02	6.78
MV efficient funds	1.02	0.99	4.18	-3.45	1.62	0.02	3.56	0.99	5.69
funds randomly picked	0.92	0.65	4.97	-2.82	2.13	1.48	8.51	0.82	4.29
Convertible arbitrage									
funds in S_U	1.07	0.97	4.83	-0.89	0.89	1.45	7.34	1.03	8.3
MV efficient funds	0.95	0.78	2.95	-0.63	0.76	0.57	4.23	0.93	8.54
funds randomly picked	0.92	0.9	3.76	-1.34	0.97	0.31	3.69	0.89	5.62
Global macro									
funds in S_U	1.01	0.91	4.71	-2.53	1.59	1.51	6.58	0.92	7.12
MV efficient funds	0.96	0.90	2.92	-1.98	1.42	0.53	5.56	0.90	8.12
funds randomly picked	0.78	0.63	3.51	-2.78	2.11	0.62	5.78	0.67	1.85

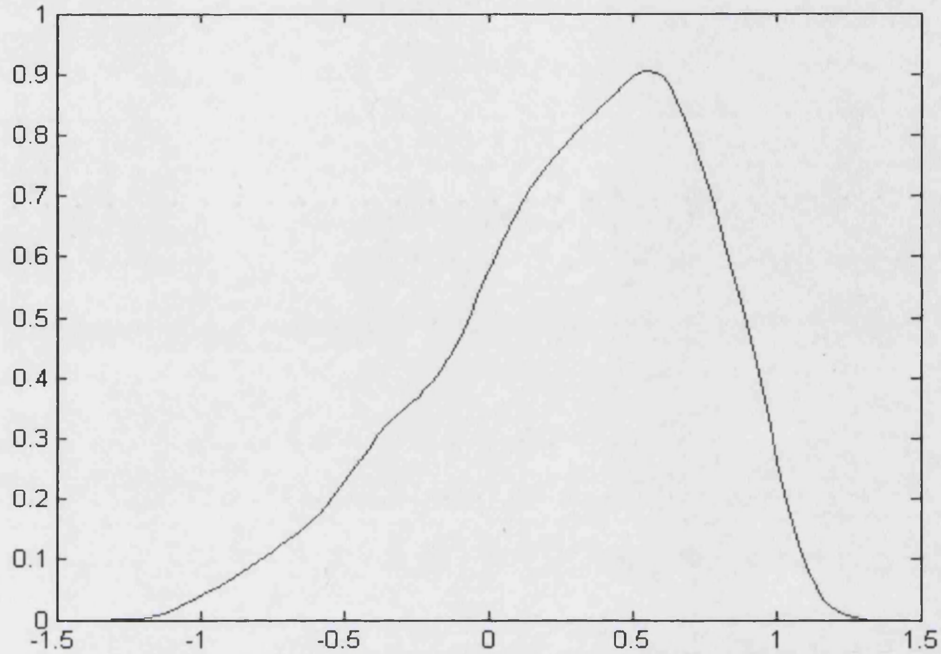

Figure 1: This figure plots cross-section estimates of alpha of hedge funds in our sample.

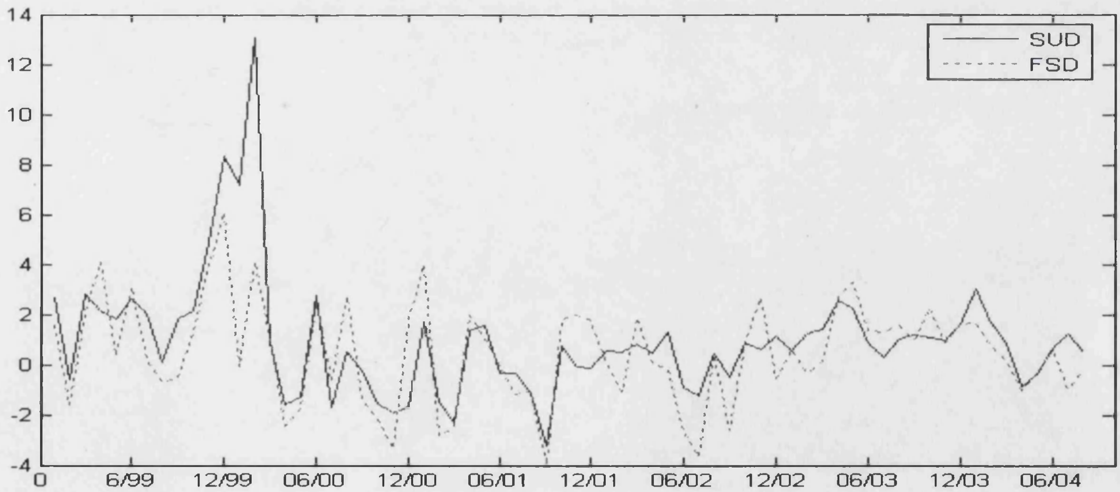

Figure 2: This figure plots return time series of portfolios of second undominated funds (SUD) and of first order dominated funds (FSD) . The portfolios are constructed on January 1 each year from 1999 to 2004.

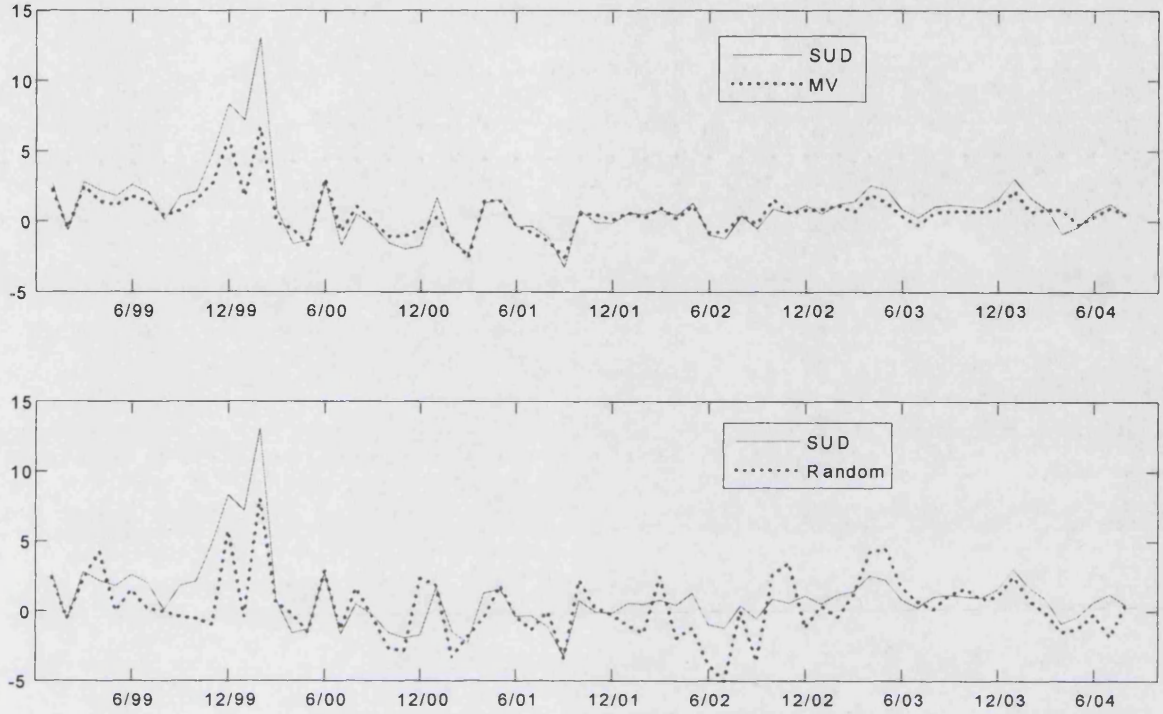

Figure 3 and 4: These two figures plot returns of portfolios of second undominated funds (SUD), mean-variance efficient funds (MV) and randomly selected funds. The portfolios are constructed on January 1 each year from 1999 to 2004.

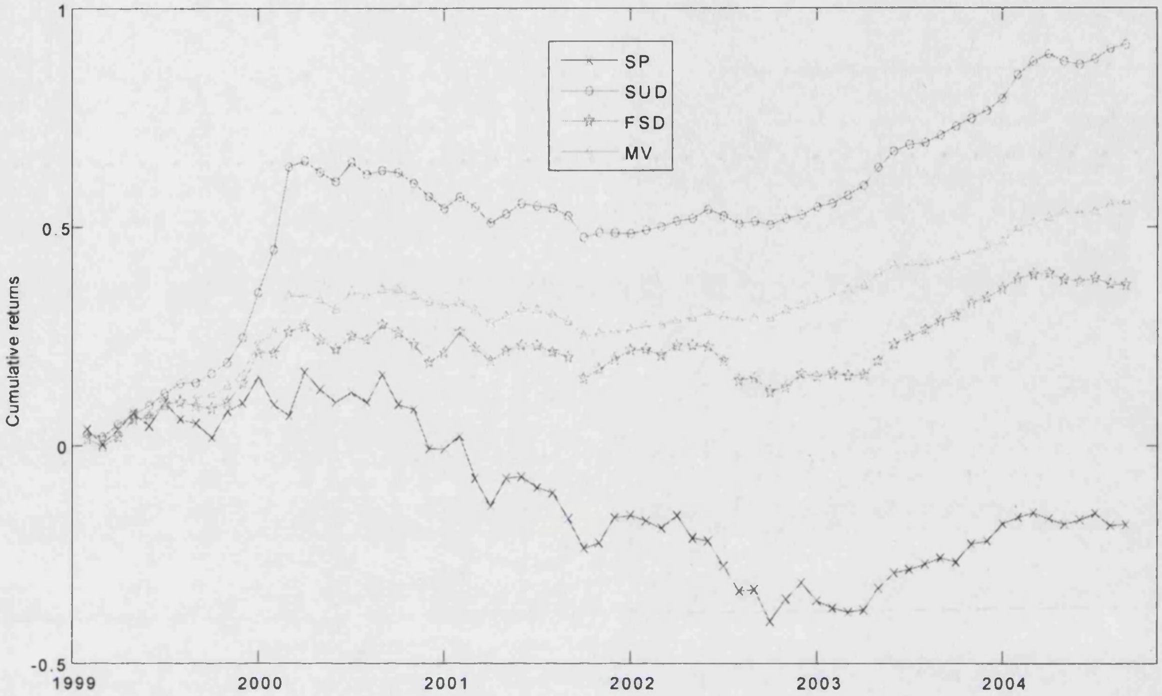

Figure 5: This figure plots cumulative returns of portfolios of second undominated funds (SUD), mean-variance efficient funds (MV), first order dominated funds (FSD). The portfolios are constructed on January 1 each year from 1999 to 2004. For comparison purpose, we also plot cumulative returns of Standard & Poor 500's stock index (01/1999 to 08/2004)

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